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16/ENG06/068
Mechanical Engineering
ENG 382

Assignment 2

(i) $f(x) = e^{-0.5x}(4-x)-2$. Given the initial guess value of $0.5 = x_0$
Maximum percentage absolute error = 1E-9. Find;
(i) Find the root of the function manually and with the aid of matlab using Newton raphson iterative method.

solution.

Manually
From Newton Raphson Iteration

$$x_{i+1} = x_i - \frac{f(x)}{f'(x)}$$

$$f(x) = e^{-0.5x}(4-x)-2$$

$f'(x) =$ $u = e^{-0.5x}$ $\quad V = 4-x$

$$\frac{du}{dx} = -0.5e^{-0.5x} \quad \frac{dv}{dx} = -1$$

From product rule;

$$U\frac{dv}{dx} + V\frac{du}{dx}$$

$$-e^{-0.5x} + (-0.5e^{-0.5x} \times 4 + x\,0.5e^{-0.5x})$$

$$-e^{-0.5x} - 2e^{-0.5x} + 0.5e^{-0.5x}$$

$$f'(x)\; x\,0.5e^{-0.5x} - 3e^{-0.5x}$$

$$e^{-0.5x}(0.5x - 3)$$

Percentage absolute error $= \left[\frac{x_{i+1} - x_i}{x_i+1}\right] \times 100$

For 1st Iteration
Let $x_i = 0.5$

$$f(x_0) = (4-0.5)e^{-0.5(0.5)} - 2$$

$$f(x_0) = 0.725802707$$

$$f'(x_0) = e^{-0.5(0.5)}\left[(0.5\times 0.3) - 3\right]$$
$$= -2.141702158$$

$$x_{i+1} = 0.5 - \frac{0.7258027407}{-2.141702153}$$

$$x_{i+1} = 0.838890606$$

$$\%\ \text{absolute error} = \left[\frac{0.838890606 - 0.5}{0.838890606}\right] \times 100\%$$

$$\%\ \text{absolute error} = 40.39747299\%$$

For 2nd Iteration

$$x_i = 0.838890606$$
$$f(x_i) = (4 - 0.838890606)\, e^{-0.5(0.838890606)} - 2$$
$$f(x_1) = 0.07814929794.$$
$$f'(x_1) = e^{-0.3\times(0.838890602)}\left[(0.5\times 0.838890606) - 3\right]$$
$$f'(x_1) = -1.696486032.$$

$$x_{i+1} = 0.838890606 - \left[\frac{0.07814929794}{-1.696486032}\right]$$

$$x_{i+1} = 0.884956003$$

$$\%\ \text{absolute error} = \left[\frac{0.884956003 - 0.838890606}{0.884956003}\right] \times 100.$$

$$\%\ \text{absolute error} = 5.205388094\%$$

For 3rd Iteration

$$x_2 = 0.884956003$$
$$f(x_2) = (4 - 0.884956003)\, e^{-0.5(0.884956003)}$$
$$f(x_2) = 0.000123657519.$$
$$f'(x_2) = e^{(-0.5\times 0.884956003)}\left[(0.5\times 0.884956003) - 3\right]$$
$$f'(x_2) = -1.643060762$$

$$x_{i+1} = 0.884956003 - \left[\frac{0.00123657519}{-1.643060762}\right]$$

$$x_{i+1} = 0.885708605$$

$$\% \text{absolute error} = \left[\frac{0.885708605 - 0.884956003}{0.885708605}\right] \times 100$$

$$\% \text{ absolute error} = 0.08497203912\%$$

For 4th Iteration

Let $x_i = 0.885708605$

$$f(x_3) = (4 - 0.885708605)e^{-0.3(0.885708605)} - 2$$

$$f(x_3) = 3.23521411 \times 10^{-7}$$

$$f'(x_3) = e^{(-0.5 \times 0.885708605)}\left[(0.5 \times 0.885708605) - 3\right]$$

$$f'(x_3) = -1.642200929$$

$$x_{i+1} = 0.885708605 - \left[\frac{3.23521411 \times 10^{-7}}{-1.642200929}\right]$$

$$x_{i+1} = 0.885708802$$

$$\% \text{ absolute error} = \left[\frac{0.885708802 - 0.885708805}{0.885708802}\right] \times 100\%$$

$$\% \text{ absolute error} = 2.224261137 \times 10^{-5}\%$$

For 5th iteration

$$x_i = 0.885708802$$

$$f(x_4) = (4 - 0.885708802)e^{-0.5(0.885708802)} - 2$$

$$f(x_4) = 7.851 \times 10^{-12}$$

$$f'(x_4) = e^{(-0.5 \times 0.885708802)}\left[(0.5 \times 0.885708802) - 3\right]$$

$$f'(x_4) = -1.642200704$$

$$x_{i+1} = 0.885708802 - \left[\frac{7.831 \times 10^{-12}}{-1.642200704}\right]$$

$$x_{i+1} = 0.885708802$$

Iterations	$x_{(i+1)}$	% error
1	0.838890206	40.39747299
2	0.884956003	5.205388094
3	0.885708605	0.08497203912
4	0.885708802	$2.224261137 \times 10^{-3}$
5	0.885708802	0