

Thank God Clinton

16/ENG06/D68

Mechanical Engineering

ENG 382

(1) In the model of a system having thermo couples measuring temperature $T(^{\circ}\text{C})$, as its different points is given by the set of expressions in eqt (1). Estimate the value of the temperature using;

(a) Gauss elimination method manually.

$$\left[\begin{array}{cccccc|c} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 & = & 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 & = & 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 & = & -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 & = & -3 \\ -3T_1 - T_2 + 2T_3 + 5T_4 + T_5 + 3T_6 & = & 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 & = & -27 \end{array} \right]$$

Solution

Forward elimination (eliminating T_1)

$$\begin{array}{l} \text{row 1} \rightarrow \\ \text{row 2} \rightarrow \\ \text{row 3} \rightarrow \\ \text{row 4} \rightarrow \\ \text{row 5} \rightarrow \\ \text{row 6} \rightarrow \end{array} \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

Multiply row 1 by 2: $2 \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \end{bmatrix}$
 $= \begin{bmatrix} 2 & 2 & -4 & 2 & 6 & -2 & 8 \end{bmatrix}$

Subtract from row 2;

$$\begin{array}{r} \begin{bmatrix} 2 & 2 & -4 & 2 & 6 & -2 & 8 \end{bmatrix} \\ - \begin{bmatrix} 2 & -1 & 1 & 2 & 1 & -3 & 20 \end{bmatrix} \\ \hline = \begin{bmatrix} 3 & -5 & 0 & 5 & 1 & -12 \end{bmatrix} \rightarrow \text{new row 1.} \end{array}$$

$$\Rightarrow 3T_2 - 5T_3 + 0T_4 + 5T_5 + T_6 = -12 \quad \text{--- (1)}$$

Subtract row 3 from row 1

$$-\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccccc|c} 0 & -2 & 1 & 2 & -1 & -2 & 19 \end{array} \right] \text{ (new row 2)}$$

$$= -2T_2 + T_3 + 2T_4 + T_5 - 2T_6 = 19 \quad \text{--- (ii)}$$

Multiply row 1 by 5.

$$5 \times \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \end{array} \right]$$

$$= \left[\begin{array}{cccccc|c} 5 & 5 & -10 & 5 & 15 & -5 & 20 \end{array} \right]$$

Subtract from row 4

$$= \left[\begin{array}{cccccc|c} 5 & 5 & -10 & 5 & 15 & -5 & 20 \end{array} \right] - \left[\begin{array}{cccccc|c} 5 & 2 & -1 & -12 & 1 & -3 \end{array} \right]$$

$$= \left[\begin{array}{cccccc|c} 3 & -9 & 6 & 13 & -6 & 23 \end{array} \right] \text{ (new row 3)}$$

$$\Rightarrow 3T_2 - 9T_3 + 6T_4 + 13T_5 - 6T_6 = 23 \quad \text{--- (iii)}$$

Multiply row 1 by 3.

$$3 \times \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \end{array} \right]$$

$$= \left[\begin{array}{cccccc|c} 3 & 3 & -6 & 3 & 9 & -3 & 12 \end{array} \right]$$

Add to row 3

$$\Rightarrow \left[\begin{array}{cccccc|c} 3 & 3 & -6 & 3 & 9 & -3 & 12 \end{array} \right] + \left[\begin{array}{cccccc|c} 3 & -1 & 2 & 3 & 13 & 16 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccccc|c} 0 & 2 & -4 & 6 & 10 & 0 & 28 \end{array} \right] \text{ (new row 4)}$$

$$\Rightarrow 2T_2 - 4T_3 + 6T_4 + 10T_5 + 0T_6 = 28 \quad \text{--- (iv)}$$

Multiply row 1 by 4

$$4 \times \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{cccccc|c} 4 & 4 & -8 & 4 & 12 & -4 & 16 \end{array} \right]$$

Subtract from row 6.

$$\left[\begin{array}{cccccc|c} 4 & 4 & -8 & 4 & 12 & -4 & 16 \end{array} \right] - \left[\begin{array}{cccccc|c} 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

$$= \left[\begin{array}{cccccc|c} 0 & 1 & -9 & 10 & 15 & -2 & 43 \end{array} \right] \text{ (new row 5)}$$

$$\Rightarrow T_2 - 9T_3 + 10T_4 + 15T_5 - 2T_6 = 43 \quad \text{--- (v)}$$

Combining eq (1-5):

$$\begin{array}{rcl}
 3T_2 - 5T_3 + 0T_4 + 5T_5 + T_6 & = & -12 \quad \text{--- (1)} \\
 -2T_2 + T_3 + 2T_4 + T_5 - 2T_6 & = & 19 \quad \text{--- (2)} \\
 3T_2 - 9T_3 + 6T_4 + 13T_5 - 6T_6 & = & 23 \quad \text{--- (3)} \\
 2T_2 - 4T_3 + 6T_4 + 10T_5 + 0T_6 & = & 28 \quad \text{--- (4)} \\
 T_2 - 9T_3 + 10T_4 + 15T_5 - 2T_6 & = & 43 \quad \text{--- (5)}
 \end{array}$$

Forward elimination (eliminating T_2):

$$\begin{array}{l}
 \text{row 1} \\
 \text{row 2} \\
 \text{row 3} \\
 \text{row 4} \\
 \text{row 5}
 \end{array}
 \left[\begin{array}{cccccc|c}
 3 & -5 & 0 & 5 & 1 & -12 \\
 -2 & 1 & 2 & 1 & -2 & 19 \\
 3 & -9 & 6 & 13 & -6 & 23 \\
 2 & -4 & 6 & 10 & 0 & 28 \\
 1 & -9 & 10 & 15 & -2 & 43
 \end{array} \right]$$

Multiply row 1 by $\frac{2}{3}$

$$\frac{2}{3} [3 \ -5 \ 0 \ 5 \ 1 \ | \ -12] = \left[2 \ -\frac{10}{3} \ 0 \ \frac{10}{3} \ \frac{2}{3} \ | \ -8 \right]$$

Add to row 2

$$\left[2 \ -\frac{10}{3} \ 0 \ \frac{10}{3} \ \frac{2}{3} \ | \ -8 \right] + [-2 \ 1 \ 2 \ 1 \ -2 \ | \ 19]$$

$$= \left[0 \ -\frac{7}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ | \ 11 \right] \text{ (new row 1)}$$

$$= -\frac{7}{3}T_3 + 2T_4 + \frac{13}{3}T_5 - \frac{4}{3}T_6 = 11 \quad \text{--- (6)}$$

Subtract row 3 from row 1

$$[3 \ -9 \ 6 \ 13 \ -6 \ | \ 23] - [3 \ -5 \ 0 \ 5 \ 1 \ | \ -12]$$

$$\Rightarrow [0 \ -4 \ 6 \ 8 \ -7 \ | \ 35] \text{ (new row 2)}$$

$$= -4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \quad \text{--- (7)}$$

Multiply row 1 by $\frac{2}{3}$

$$\frac{2}{3} [3 \ -5 \ 0 \ 3 \ 1 \ | \ -12] = \left[2 \ -\frac{10}{3} \ 0 \ \frac{10}{3} \ \frac{2}{3} \ | \ -8 \right]$$

Subtract from row (4)

$$[2 \ -4 \ 6 \ 10 \ 0 \ | \ 28] - \left[2 \ -\frac{10}{3} \ 0 \ \frac{10}{3} \ \frac{2}{3} \ | \ -8 \right]$$

$$= [0 \ -\frac{2}{3} \ 6 \ \frac{20}{3} \ -\frac{2}{3} \ | \ 36] \text{ (new row 3)}$$

$$\Rightarrow -\frac{2}{3}T_3 + 6T_4 + \frac{20}{3}T_5 - \frac{2}{3}T_6 = 36 \quad \text{--- (8)}$$

Multiply row 1 by $\frac{1}{3}$

$$\frac{1}{3} \times [3 \ -5 \ 0 \ 5 \ 1 \ | \ -12] = [1 \ -\frac{5}{3} \ 0 \ \frac{5}{3} \ \frac{1}{3} \ | \ -4]$$

Subtract from row 5

$$[1 \ -9 \ 10 \ 15 \ -2 \ | \ 43] - [1 \ -\frac{5}{3} \ 0 \ \frac{5}{3} \ \frac{1}{3} \ | \ -4]$$

$$= [-\frac{22}{3} \ 10 \ \frac{40}{3} \ -\frac{7}{3} \ | \ 47] \text{ (new row 4)}$$

$$= \frac{22}{3}T_3 + 10T_4 + \frac{40}{3}T_5 - \frac{7}{3}T_6 = 47 \quad \text{--- (9)}$$

Combining eqt (6-9)

$$\frac{7}{3}T_3 + 2T_4 + \frac{13}{3}T_5 - \frac{4}{3}T_6 = 11 \quad \text{--- (6)}$$

$$-4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \quad \text{--- (7)}$$

$$-\frac{2}{3}T_3 + 6T_4 + \frac{20}{3}T_5 + \frac{2}{3}T_6 = 36 \quad \text{--- (8)}$$

$$-\frac{22}{3}T_3 + 10T_4 + \frac{40}{3}T_5 - \frac{7}{3}T_6 = 47 \quad \text{--- (9)}$$

Forward elimination (eliminating T_3)

row 1 -	$[-\frac{7}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ \ 11]$
row 2	$[-4 \ 6 \ 8 \ -7 \ \ 35]$
row 3	$[-\frac{2}{3} \ 6 \ \frac{20}{3} \ -\frac{2}{3} \ \ 36]$
row 4	$[-\frac{22}{3} \ 10 \ \frac{40}{3} \ -\frac{7}{3} \ \ 47]$

Multiply row 1 by $\frac{12}{7}$

$$\frac{12}{7} [-\frac{7}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ | \ 11] = [-4 \ \frac{24}{7} \ \frac{52}{7} \ -\frac{16}{7} \ | \ \frac{132}{7}]$$

Eqt 2 to be subtracted from this.

$$\Rightarrow [-4 \ \frac{24}{7} \ \frac{52}{7} \ -\frac{16}{7} \ | \ \frac{132}{7}] - [-4 \ 6 \ 8 \ -7 \ | \ 35]$$

$$\text{new row 1} \quad [0 \ -\frac{18}{7} \ -\frac{4}{7} \ \frac{33}{7} \ | \ -\frac{113}{7}]$$

$$\Rightarrow -\frac{18}{7}T_4 - \frac{4}{7}T_5 + \frac{33}{7}T_6 = -\frac{113}{7} \quad \text{--- (10)}$$

Multiply row 1 by $\frac{2}{7}$

$$\frac{2}{7} [-\frac{7}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ | \ 11] = [-\frac{2}{3} \ \frac{4}{7} \ \frac{26}{21} \ -\frac{8}{21} \ | \ \frac{22}{7}]$$

Subtract row 3 from above;

$$[-\frac{2}{3} \ \frac{4}{7} \ \frac{26}{21} \ -\frac{8}{21} \ | \ \frac{22}{7}] - [-\frac{2}{3} \ 6 \ \frac{20}{3} \ -\frac{2}{3} \ | \ 36]$$

$$= [0 \ -\frac{38}{7} \ -\frac{38}{7} \ \frac{2}{7} \ | \ -\frac{230}{7}] \text{ (new row 2)}$$

$$= -\frac{38}{7}T_4 - \frac{38}{7}T_5 + \frac{2}{7}T_6 = -\frac{230}{7} \quad \text{--- (11)}$$

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Multiply row 1 by $22/7$

$$\begin{aligned} \frac{22}{7} \left[\begin{array}{ccc|c} -7/3 & 2 & 13/3 & -4/3 \\ -22/3 & 44/7 & 286/21 & -88/21 \\ 0 & -26/7 & 2/7 & -13/7 \end{array} \right] &= \left[\begin{array}{ccc|c} -22/3 & 44/7 & 286/21 & -88/21 \\ -22/3 & 10 & 40/3 & -7/3 \\ 0 & -26/7 & 2/7 & -13/7 \end{array} \right] \\ &= \left[\begin{array}{ccc|c} 0 & -26/7 & 2/7 & -13/7 \\ -26/7 T_4 + 2/7 T_5 - 13/7 T_6 & & & -87/7 \end{array} \right] \text{ (new row 3)} \\ &= -26/7 T_4 + 2/7 T_5 - 13/7 T_6 = -87/7 \quad \text{--- (12)} \end{aligned}$$

Combining eqs (10-12)

$$\begin{aligned} -18/7 T_4 - 4/7 T_5 + 33/7 T_6 &= -113/7 \quad \text{--- (10)} \\ -38/7 T_4 - 380/7 T_5 + 2/7 T_6 &= -230/7 \quad \text{--- (11)} \\ -26/7 T_4 + 2/7 T_5 - 13/7 T_6 &= -87/7 \quad \text{--- (12)} \end{aligned}$$

Forward elimination (eliminating T_4)

$$\begin{aligned} \left[\begin{array}{ccc|c} -18/7 & -4/7 & 33/7 & -113/7 \\ -38/7 & -380/7 & 2/7 & -230/7 \\ -26/7 & 2/7 & -13/7 & -87/7 \end{array} \right] & \begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{array} \end{aligned}$$

Multiply row 1 by $19/9$

$$\frac{19}{9} \times \left[\begin{array}{ccc|c} -18/7 & -4/7 & 33/7 & -113/7 \end{array} \right] = \left[\begin{array}{ccc|c} -38/7 & -76/63 & 209/21 & -2147/63 \end{array} \right]$$

Subtract row 2 from the above

$$\begin{aligned} \left[\begin{array}{ccc|c} -38/7 & -76/63 & 209/21 & -2147/63 \\ -38/7 & -380/7 & 2/7 & -230/7 \end{array} \right] &= \left[\begin{array}{ccc|c} 0 & 38/9 & 29/3 & -11/9 \end{array} \right] \\ &= 38/9 T_5 + 29/3 T_6 = -11/9 \quad \text{--- (13)} \end{aligned}$$

Multiply row 1 by $13/9$

$$\frac{13}{9} \times \left[\begin{array}{ccc|c} -18/7 & -4/7 & 33/7 & -113/7 \end{array} \right] = \left[\begin{array}{ccc|c} -26/7 & -52/63 & 143/21 & -1489/63 \end{array} \right]$$

Subtract row 3 from the above;

$$\begin{bmatrix} -26/7 & -52/63 & 143/21 & -1469/63 \end{bmatrix} - \begin{bmatrix} 26/7 & 7/7 & -13/7 & -87/7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -10/9 & 26/3 & -98/9 \end{bmatrix}$$

$$\therefore -10/9 T_5 + 26/3 T_6 = 98/9 \quad \text{--- (14)}$$

Combining eqt (13) and (14)

$$38/9 T_5 + 29/3 T_6 = -11/9 \quad \text{--- (13)}$$

$$-10/9 T_5 + 26/3 T_6 = 98/9 \quad \text{--- (14)}$$

From eqt (13) make T_6 subject of formula

$$T_6 = \left(\frac{-11}{9} - \frac{38}{9} T_5 \right) \times \frac{3}{29} \quad \text{--- (15)}$$

Backward Substitution

Substitute T_6 in eqt (14)

$$\frac{-10}{9} T_5 + \frac{26}{3} \left(\frac{-11}{9} - \frac{38}{9} T_5 \right) \frac{3}{29} = \frac{-98}{9}$$

$$\frac{-10}{9} T_5 \left(\frac{-286}{27} - \frac{988}{27} T_5 \right) \frac{3}{29} = \frac{-98}{9}$$

$$-10/9 T_5 - \frac{286}{261} - \frac{988}{261} T_5 = -98/9$$

$$-\frac{142}{29} T_5 = \frac{-284}{29}$$

$$T_5 = -284/142 = 2$$

Substitute T_5 in eqt 15; $T_6 = \left(\frac{-11}{9} - \frac{38 \times 2}{9} \right) \times \frac{3}{29}$

$$\left(\frac{-4}{9} - \frac{76}{9} \right) \times \frac{3}{29}$$

$$T_6 = -1$$

Substitute T_5 and T_6 in eqt (10)

$$-18/7 T_4 - 8/7 - 33/7 = -113/7$$

$$-18/7 T_4 = -113/7 + 33/7 + 8/7$$

$$-18/7 T_4 = -72/7$$

$$T_4 = \frac{-72}{7} \times \frac{-7}{18}$$

$$T_4 = 4$$

Backward Substitution.

Substitute T_4 , T_5 and T_6 in eqt (7)

From eqt (7)

$$-4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \quad \text{--- (7)}$$

$$\text{Where; } T_4 = 4$$

$$T_5 = 2$$

$$T_6 = -1$$

$$-4T_3 + 6(4) + 8(2) - 7(-1) = 35$$

$$-4T_3 + 24 + 16 + 7 = 35$$

$$-4T_3 = 35 - 47$$

$$-4T_3 = -12$$

$$T_3 = 3$$

From eqt (1)

$$3T_2 - 5T_3 + 5T_5 + T_6 = -12 \quad \text{--- (1)}$$

Substitute T_3 , T_5 and T_6 in eqt (1)

$$3T_2 - 5(3) + 5(2) - 1 = -12$$

$$3T_2 = -12 + 15 - 10 + 1$$

$$3T_2 = -6$$

$$T_2 = -2$$

From the initial equations

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

Substitute T_2 , T_3 , T_4 , T_5 and T_6

$$T_1 + (-2) - 2(3) + 4 + 3(2) + 1 = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 = 4 + 2 + 6 - 4 - 6 - 1$$

$$T_1 = 1$$

$$T_1 = 1$$

$$T_6 = -1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$