Assignment II   27/02/2019

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If the maximum percentage absolute
error is desired to be 1E-9,
using the Newton-Raphson iteration
method and initial guess value of 0.5,
find the root of the function given
in equation (1.1)

a) manually

b) with the aid of MATLAB.

$$f(x) = e^{-0.5x}(4-x) - 2$$

From Newton Raphson's equation

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = e^{0.5x}(4-x) - 2$$

$$\overset{\downarrow}{u} \quad \overset{\downarrow}{v}$$

Using the product rule

$$\frac{f(x)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{df(x)}{dx} = A$$

$$\frac{dv}{dx} = (-1)$$

$$\frac{du}{dx} = -0.5e^{-0.5x}$$

$$\frac{df(x)}{dx} = e^{-0.5x}\left[-1\right] + \left((4-x) - ①\right) \times (-0.5e^{-0.5x})$$

$$f'x = -e^{-0.5x} + (4-x)(-0.5e^{-0.5x})$$

$$f'x = -e^{-0.5x} + 0.5e^{-0.5x}(x-4)$$

$$f'(x) = 0.5e^{-0.5x}(x-4) - e^{-0.5x}$$

$$f(x) = e^{-0.5x}(4-x) - 2$$

At $i = 0$

x	i	Error
0.5	0	
1		0
0.83889	0.83889	40.9%
0.884955	2	-5.2%
0.883206	3	0.08459
0.883162	4	9.9355 × 10^{-4}%
0.8857045	5	2.8113 × 10^{-4}%
0.8857009	6	5.64513 × 10^{-4}%

Solution:

$$x_1 = 0.5 + \frac{e^{-0.5(0.5)}(4-0.5) - 2}{0.5e^{-0.5x}(x-4) - e^{0.5x}}$$

$$x_1 = 0.5 + \frac{e^{-0.5(0.5)}(4-0.5) - 2}{0.5e^{-0.5(0.5)}(0.5-4) - e^{-0.5(0.5)}}$$

$$x_1 = 0.5 + \frac{0.726}{2.141}$$

$$x(1) = 0.83889$$

$$x_2 = 0.83889 - \frac{e^{-0.5 \times 0.83889}(4-0.83889)-2}{0.5e^{-0.5 \times 0.83889}(0.83889-4)-e^{-0.5 \times (0.83889)}}$$

$$x_2 = 0.83889 + 0.04606598791$$

$$= 0.884955$$

$$x_3 = 0.884955 - \frac{e^{-0.5 \times 0.884956}(4-0.884955)-2}{0.5e^{-0.5 \times 0.884956}(0.884956-4)-e^{-0.5 \times (0.884956)}}$$

$$x_3 = 0.884955 - \frac{e^{-0.5 \times 0.8842}(4-0.8842)-2}{0.5e^{-0.5 \times 0.8842}(0.8842-4)-e^{-0.5 \times (0.8842)}}$$

$$x_3 = 0.884955 + 7.478607 \times 10^{-4}$$

$$= 0.8857036$$

$$x_4 = 0.8857036 + \frac{e^{-0.5 \times 0.8842}(4-0.8842)-2}{0.5e^{-0.5 \times 0.8842}(0.8842-4)-e^{-0.5 \times (0.8842)}}$$

$$x_4 = 0.8857036 + 8.80178 \times 10^{-6}$$

$$= 0.8857912402$$

$$x_5 = 0.88289 - \frac{e^{-0.5 \times 0.8827}(4-0.8827)-2}{0.5e^{-0.5 \times 0.8827}(0.8827-4)-}$$

$$x_5 = 0.88269 + \frac{4.8913 \times 768 e^{-0.5 \times (0.8827)}}{2.015635 \times 10^{-6}}$$

$$x_5 = 0.8857484$$

$$x_6 = 0.88857 - \frac{e^{-0.5 \times 0.8857}(4-0.8857)-2}{0.5e^{-0.5 \times 0.8857} \times (0.8857-4)-e^{-0.5 \times 0}}$$

$$x_6 = 0.8857 - \frac{6.199904903}{8.8019083857 \times 10^{-6}}$$

$$x_6 = 0.8857087$$

$$\epsilon_q = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%$$

$$\epsilon_{q_1} = \left| \frac{0.83889 - 0.5}{0.83889} \right| \times 100\%$$

$$\epsilon_{q_1} = 40.49\%$$

$$\epsilon_{q_2} = \left| \frac{0.889453 - 0.83889}{0.889455} \right| \times 100\%$$

$$\epsilon_{q_2} = 5.2053\%$$

$$\epsilon_{q_3} = \left| \frac{0.8857036 - 0.889455}{0.8857636} \right| \times 100$$

$$\epsilon_{q_3} = 0.084529\%$$

$$\epsilon_{q_4} = \left| \frac{0.8857124 - 0.8857036}{0.8857124} \right| \times 100$$

$$\epsilon_{q_4} = 9.93355 \times 10^{-4}\%$$

$$\epsilon_{q_5} = \left| \frac{0.8857149 - 0.8857124}{0.8857149} \right| \times 100$$

$$\epsilon_{q_5} = 2.8113 \times 10^{-4}\%$$

$$\epsilon_{q_6} = \left| \frac{0.8857119 - 0.8857114}{0.8857119} \right| \times 100$$

$$\epsilon_{q_6} = 5.6451312 \times 10^{-4}\%$$

$x_1 \,\&\, 1$ $x_4 = 4$
$x_2 = -2$ $x_5 = 2$
$x_3 = 3$ $x_6 = -1$

$$
\begin{bmatrix}
1 & -2 & 1 & 3 & -1 \\
2 & -1 & 1 & 2 & -3 \\
1 & 3 & -3 & -1 & 2 \\
5 & 2 & -1 & 2 & 1 \\
-3 & -1 & 2 & 3 & 3 \\
4 & 3 & 1 & -6 & -3 & -2
\end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}
=
\begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}
$$

$$
\begin{bmatrix}
1 & 1 & 2 & 1 & 3 & -1 \\
2-\tfrac{2}{7}y) & -1\tfrac{2}{7}(-1) & 1\tfrac{9}{7}(-2) & 2\tfrac{2}{7}
\end{bmatrix}
$$

$$
\begin{bmatrix}
1 & -2 & 1 & 3 & -1
\end{bmatrix}
$$

$2-\tfrac{2}{7}(1)$ $-1-\tfrac{2}{7}(-2)$ $2-\tfrac{2}{7}(1)$ $1-\tfrac{2}{7}(3)$ $-3-\tfrac{2}{7}(-1)$

$1-\tfrac{1}{7}(1)$ $3-\tfrac{1}{7}(-2)$ $-3-\tfrac{1}{7}(-2)$ $-1-\tfrac{1}{7}(1)$ $2-\tfrac{1}{7}(3)$ $1-\tfrac{1}{7}(-1)$

$5-\tfrac{5}{7}(1)$ $2-\tfrac{5}{7}(-2)$ $-1-\tfrac{5}{7}(-2)$ $-1+\tfrac{5}{7}(1)$ $2-\tfrac{5}{7}(3)$ $1-\tfrac{5}{7}(-1)$

$-3+\tfrac{3}{7}(1)$ $-1+\tfrac{3}{7}(-2)$ $2+\tfrac{3}{7}(-2)$ $3+\tfrac{3}{7}(1)$ $1+\tfrac{3}{7}(3)$ $3+\tfrac{3}{7}(-1)$

$4-\tfrac{4}{7}(1)$ $3-\tfrac{4}{7}(1)$ $1-\tfrac{4}{7}(-2)$ $-6-\tfrac{4}{7}(1)$ $-3-\tfrac{4}{7}(3)$ $-2-\tfrac{4}{7}(-4)$

1	1	-2	1	3	-1	4
0	-3	5	0	-5	-1	12
0	2	-1	-2	-1	2	-19
0	-3	9	-6	-13	6	-23
0	2 = -4	6	10	0		-28
0	-1	9	-16	-15		-43

1	1	-2	1	3	-1
0	$-3 + \frac{3}{7}(1)$	$5 + \frac{3}{7}(-2)$	$0 + \frac{3}{7}(1)$		

1	1	-2	1	3	-1	
0	-3	5	0	-5	-1	
	2					

1	1	-2	1	3	-1	4
0	$2 + \frac{2}{3}(-3)$	$-1 + \frac{2}{3}(5)$	$-2 + \frac{2}{3}(0)$	$-1 + \frac{2}{3}(5)$	$2 + \frac{2}{3}(-1)$	$-19 + \frac{2}{3}(1)$
0	$-3 - \frac{3}{3}(-3)$	$9 - \frac{3}{3}(5)$	$-6 - 0$	$-13 + 5$	$6 + 1$	$-23 - (12)$
0	$2 + \frac{2}{3}(-3)$	$-4 + \frac{2}{3}(5)$	$6 - 0$	$10 + \frac{2}{3}(-5)$	$0 + \frac{2}{3}(-1)$	$28 + \frac{2}{3}(12)$
0	$-1 \frac{3}{3}(-3)$	$9 - \frac{1}{3}(5)$	$-10 - 0$	$-15 - \frac{1}{3}(-5)$	$2 - \frac{1}{3}(-1)$	$-43 - \frac{1}{3}(12)$

$$1 \quad -2 \quad | \quad 3 \quad -1 \quad 4$$
$$0 \quad -3 \quad 5 \quad 0 \quad -5 \quad -1 \quad 12$$
$$0 \quad 0 \quad \tfrac{7}{3} \quad -2 \quad -\tfrac{13}{3} \quad \tfrac{4}{3} \quad -11$$
$$0 \quad 0 \quad 4 \quad -6 \quad -8 \quad 7 \quad -35$$
$$0 \quad 0 \quad -\tfrac{2}{3} \quad 6 \quad \tfrac{20}{3} \quad -\tfrac{2}{3} \quad 36$$
$$0 \quad 0 \quad -\tfrac{22}{3} \quad -10 \quad -\tfrac{40}{3} \quad \tfrac{7}{3} \quad -47$$

$$1 \quad -2 \quad 1 \quad 3 \quad -1 \quad 4$$
$$0 \quad -3 \quad 5 \quad 0 \quad -5 \quad -1 \quad 12$$
$$0 \quad 0 \quad \tfrac{7}{3} \quad -2 \quad -\tfrac{13}{3} \quad \tfrac{4}{3} \quad -11$$

$$0 \quad 0 \quad 4 - \tfrac{12}{7}(\tfrac{7}{3}) \quad -6 - \tfrac{12}{7}(-2) \quad -8 - \tfrac{12}{7}(-\tfrac{13}{3}) \quad 7 - \tfrac{12}{7}(\tfrac{4}{3}) \quad -35 - \tfrac{12}{7}(-11)$$
$$0 \quad 0 \quad -\tfrac{2}{3} + \tfrac{2}{7}(\tfrac{7}{3}) \quad 6 + \tfrac{2}{7}(-2) \quad \tfrac{20}{3} + \tfrac{2}{7}(-\tfrac{13}{3}) \quad -\tfrac{2}{3} + \tfrac{2}{7}(\tfrac{4}{3}) \quad 36 + \tfrac{2}{7}(-11)$$
$$0 \quad 0 \quad \tfrac{22}{3} - \tfrac{22}{7}(\tfrac{7}{3}) \quad -10 - \tfrac{22}{7}(-2) \quad -\tfrac{40}{3} - \tfrac{22}{7}(-\tfrac{13}{3}) \quad \tfrac{7}{3} - \tfrac{22}{7}(\tfrac{4}{3}) \quad -47 - \tfrac{22}{7}(-11)$$

$$1 \quad -2 \quad 1 \quad 3 \quad -1 \quad 4$$
$$0 \quad -3 \quad 5 \quad 0 \quad -5 \quad -1 \quad 12$$
$$0 \quad 0 \quad \tfrac{7}{3} \quad -2 \quad -\tfrac{13}{3} \quad \tfrac{4}{3} \quad -11$$
$$0 \quad 0 \quad 0 \quad -2 \quad -\tfrac{13}{3} \quad \tfrac{4}{3} \quad -11$$
$$0 \quad 0 \quad 0 \quad -\tfrac{18}{7} \quad -\tfrac{4}{7} \quad \tfrac{33}{7} \quad -\tfrac{113}{7}$$
$$0 \quad 0 \quad 0 \quad \tfrac{38}{7} \quad \tfrac{34}{7} \quad -\tfrac{2}{7} \quad \tfrac{230}{7}$$
$$0 \quad 0 \quad 0 \quad -\tfrac{26}{7} \quad \tfrac{2}{7} \quad -\tfrac{13}{7} \quad -\tfrac{87}{7}$$

$$
\begin{array}{cccccc}
+1 & 1 & -2 & 1 & 3 & -1 \\
0 & 1-3 & 5 & 0 & -5 & -1 \\
0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\
0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\
0 & 0 & 0 & 38/7 & 38/7 & -2/7 \\
0 & 0 & 0 & -26/7 & 2/7 & -13/7
\end{array}
$$

$$
\begin{array}{cccccc|c}
1 & 1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\
0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\
0 & 0 & 0 & 38/7 & 34/7 & -2/7 & 230/7 \\
0 & 0 & 0 & -26/7 & 2/7 & -13/7 & -87/7
\end{array}
$$

$$
\begin{array}{cccccc|c}
1 & 1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\
0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\
\end{array}
$$

$$
\frac{35}{7}+\frac{19}{9}\left(-\frac{18}{7}\right) \quad \frac{15}{7}\left(\frac{19}{9}\right)\left(\frac{7}{9}\right) \quad -\frac{2}{7}+\frac{19}{9}\left(\frac{33}{7}\right) \quad -\frac{11}{9}
$$

$$
\frac{-26}{7}-\frac{13}{9}\left(\frac{-18}{7}\right) \quad \frac{2}{7}-\frac{13}{9}\left(\frac{-4}{7}\right) \quad \frac{-13}{7}-\frac{13}{9}\left(\frac{33}{7}\right) \quad \frac{98}{9}
$$

$$\begin{array}{ccccccc}
1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\
0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\
0 & 0 & 0 & 0 & 34/9 & 29/3 & -11/9 \\
0 & 0 & 0 & 0 & 0 & 10/9 & -26/3 & 98/9
\end{array}$$

$$\begin{array}{cccccc}
x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\
1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\
0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\
0 & 0 & 0 & 0 & 34/9 & 29/3 & -11/9 \\
0 & 0 & 0 & 0 & 0 & -26/3 & 98/9
\end{array}$$

$$\frac{10}{9} - \frac{19}{38}\left(\frac{38}{9}\right) \quad \frac{-26}{3} - \frac{10}{38}\left(\frac{29}{3}\right)$$

$$0 \quad \frac{-213}{19} \quad \frac{98}{9} - \frac{10}{36}\left(\frac{-11}{9}\right)$$

$$0 + \frac{-213}{19} x_6 = \frac{213}{19}$$

$$x_6 = \frac{213}{19} \cdot \frac{19}{-213}$$

$$x_6 = -1$$

$$x_5 = 2$$

$$3x_5 = -11/9 + 29/3$$

$$x_5 = 26/9 \times 9/38 =$$

$$-18/7 x_4 + (-4/7)(2) + \dfrac{33}{7}(-1) = -113/7$$

$$-18/7 x_4 + (-41/7) = -113/7$$

$$-18/7 x_4 = -113/7 + 41/7$$

$$-18/7 x_4 = -72/7$$

$$x_4 = 4$$

$$\tfrac{1}{3} x_3 + (-2)(4) + \dfrac{-13}{3}(2) + 4/3(-1) = -11$$

$$\tfrac{1}{3} x_3 - 18 = -11$$

$$\tfrac{1}{3} x_3 = -11 + 18$$

$$\tfrac{1}{3} x_3 = 7$$

$$x_3 \cdot \tfrac{1}{x} \cdot \tfrac{3}{x} = 3$$

$$x_3 = 3$$

$-3x_2 + 5(3) + 0 + (-5)(2) + (-1)(-1) = 12$

$-3x_2 + 6 = 12$

$-3x_2 = 12 - 6$

$-3x_2 = 6$

$x_2 = -2$

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$x_1 + (-2) + (-2)(+3) + (1)(4) + 3(2) + (-1)(-1) = 9$

$x_1 - 2 \neq 6 + 4 + 6 + 1 = 4$

$x_1 + B = 4$

$x_1 = 4 - 3 = 1$

$x_1 = 1$

———

$-3x_2 + 5(3) + 0 + (-5)(2) + (-1)(-1) = 12$

$-3x_2 + 6 = 12$

$-3x_2 = 12 - 6$

$-3x_2 = 6$

$x_2 = -2$

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$x_1 + (-2) + (-2)(+3) + (1)(4) + 3(2) + (-1)(-1)$

$x_1 - 2 \neq 6 + 4 + 6 + 1 = 4$

$x_1 \neq 8 = 4$

$x_1 = 4 - 3 = 1$

$x_1 = 1$

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