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16/ENGG06/001

Mechanical Engineering

Eng 382

1) In the model of a system having thermo coupled measuring temp $T(^{\circ}\text{C})$ at its different points is given by the set of expressions in eqt (1). Estimate the value of the temperature using!

a) Gauss elimination method manually;

$$\begin{cases} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 + T_4 - 6T_5 - 3T_6 = -27 \end{cases}$$

Solution.

forward Elimination (eliminating T_1).

$$\begin{array}{l} \text{row 1} \rightarrow \\ \text{row 2} \rightarrow \\ \text{row 3} \rightarrow \\ \text{row 4} \rightarrow \\ \text{row 5} \rightarrow \\ \text{row 6} \rightarrow \end{array} \left[\begin{array}{cccccc} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{array} \right] \begin{array}{l} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{array}$$

$$\begin{aligned} \text{multiply row 1 by 2: } & 2[1 \ 1 \ -2 \ 1 \ 3 \ -1 \ 4] \\ & = [2 \ 2 \ -4 \ 2 \ 6 \ -2 \ 8] \end{aligned}$$

Subtract from row 2;

$$\begin{aligned} & [2 \ 2 \ -4 \ 2 \ 6 \ -2 \ 8] \\ & - [2 \ -1 \ 1 \ 2 \ 1 \ -3 \ 20] \\ & = [3 \ -5 \ 0 \ 5 \ 1 \ -12] \rightarrow \text{new row 1.} \\ & \Rightarrow 3T_2 - 5T_3 + 0T_4 + 5T_5 + T_6 = -12 \quad \text{--- (1)} \end{aligned}$$

Subtract row 3 from row 1

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -2 & 1 & 2 & 1 & -2 & 19 \end{bmatrix} \text{ [new row 2]}$$

$$= -2T_2 + T_3 + 2T_4 + T_5 - 2T_6 = 19 \quad \text{--- (ii)}$$

Multiply row 1 by 3.

$$3 \times \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -6 & 3 & 9 & -3 & 12 \end{bmatrix}$$

Subtract from row 4

$$= \begin{bmatrix} 3 & 3 & -6 & 3 & 9 & -3 & 12 \end{bmatrix} - \begin{bmatrix} 3 & 2 & -1 & -1 & 2 & 1 & -5 \end{bmatrix}$$

$$\Rightarrow 3T_2 - 9T_3 + 6T_4 + 13T_5 - 6T_6 = 23 \quad \text{--- (iii)}$$

Multiply row 1 by 5.

$$5 \times \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 5 & -10 & 5 & 15 & -5 & 20 \end{bmatrix}$$

Add to row 3

$$\Rightarrow \begin{bmatrix} 3 & 3 & -6 & 3 & 9 & -3 & 12 \end{bmatrix} + \begin{bmatrix} 5 & -1 & 2 & 3 & 13 & 1 & 16 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2 & -4 & 6 & 10 & 0 & 28 \end{bmatrix} \text{ (new row 4)}$$

$$\Rightarrow 2T_2 - 4T_3 + 6T_4 + 10T_5 + 0T_6 = 28 \quad \text{--- (iv)}$$

Multiply row 1 by 4

$$4 \times \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 4 & -8 & 4 & 12 & -4 & 16 \end{bmatrix}$$

Subtract from row 6.

$$\begin{bmatrix} 4 & 4 & -8 & 4 & 12 & -4 & 16 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 1 & -6 & 3 & -2 & 29 \end{bmatrix}$$

$$\frac{1}{2} \times 2T_2 + 10T_4$$

$$\begin{bmatrix} 0 & 1 & -1 & 10 & 15 & -2 & 48 \end{bmatrix} \text{ (new row 5)}$$

$$\Rightarrow T_2 - 9T_3 + 10T_4 + 15T_5 - 2T_6 = 43 \quad \text{--- (v)}$$

$$\text{combining eqt (1-5)}; 3T_2 - 5T_3 + 0T_4 + 5T_5 + T_6 = -12 \quad (1)$$

$$-2T_2 + T_3 + 2T_4 + T_5 - 2T_6 = 19 \quad (2)$$

$$3T_2 - 9T_3 + 6T_4 + 15T_5 - 6T_6 = 23 \quad (3)$$

$$2T_2 - 4T_3 + 6T_4 + 10T_5 + 0T_6 = 28 \quad (4)$$

$$T_2 - 4T_3 + 10T_4 + 15T_5 - 2T_6 = 43 \quad (5)$$

forward elimination (eliminating T_2)

$$\begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \\ \text{row 4} \\ \text{row 5} \end{array} \left[\begin{array}{ccccc|c} 3 & -5 & 0 & 5 & 1 & -12 \\ -2 & 1 & 2 & 1 & -2 & 19 \\ 3 & -9 & 6 & 15 & -6 & 23 \\ 2 & -4 & 6 & 10 & 0 & 28 \\ 1 & -4 & 10 & 15 & -2 & 43 \end{array} \right]$$

multiply row 1 by $\frac{2}{3}$

$$\frac{2}{3} [3 \ -5 \ 0 \ 5 \ 1 \ | \ -12] = \left[2 \ -\frac{10}{3} \ 0 \ \frac{10}{3} \ \frac{2}{3} \ | \ -8 \right]$$

Add to row 2

$$\left[2 \ -\frac{10}{3} \ 0 \ \frac{10}{3} \ \frac{2}{3} \ | \ -8 \right] + \left[-2 \ 1 \ 2 \ 1 \ -2 \ | \ 19 \right]$$

$$= \left[0 \ -\frac{7}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ | \ 11 \right] \text{ (new row 1)}$$

$$= -\frac{7}{3}T_3 + 2T_4 + \frac{13}{3}T_5 - \frac{4}{3}T_6 = 11 \quad (6)$$

Subtract row 3 from 1

$$\left[3 \ -9 \ 6 \ 13 \ -6 \ | \ 23 \right] - \left[3 \ -5 \ 0 \ 5 \ 1 \ | \ -12 \right]$$

$$\Rightarrow \left[0 \ -4 \ 6 \ 8 \ -7 \ | \ 35 \right] \text{ (new row 2)}$$

$$= -4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \quad (7)$$

multiply row 1 by $\frac{2}{3}$

$$\frac{2}{3} [3 \ -5 \ 0 \ 3 \ 1 \ | \ -12] = \left[2 \ -\frac{10}{3} \ 0 \ \frac{10}{3} \ \frac{2}{3} \ | \ -8 \right]$$

Subtract from row (4)

$$\left[2 \ -4 \ 6 \ 10 \ 0 \ | \ 28 \right] - \left[2 \ -\frac{10}{3} \ 0 \ \frac{10}{3} \ \frac{2}{3} \ | \ -8 \right]$$

$$= \left[0 \ -\frac{2}{3} \ 6 \ \frac{20}{3} \ -\frac{2}{3} \ | \ 36 \right] \text{ (new row 3)}$$

$$\Rightarrow -\frac{2}{3}T_3 + 6T_4 + \frac{20}{3}T_5 - \frac{2}{3}T_6 = 36 \quad (8)$$

multiply row 1 by $\frac{1}{3}$

$$\frac{1}{3} \times [3 \ -5 \ 0 \ 5 \ 1 \ -12] = [1 \ -\frac{5}{3} \ 0 \ \frac{5}{3} \ \frac{1}{3} \ -4]$$

Subtract from row 5.

$$[1 \ -9 \ 10 \ 15 \ -2/43] - [1 \ -\frac{5}{3} \ 0 \ \frac{5}{3} \ \frac{1}{3} \ -4]$$

$$= [-\frac{22}{3} \ 10 \ \frac{40}{3} \ -\frac{7}{3} \ 47] \text{ (new row 4)}$$

$$= \frac{22}{3}T_3 + 10T_4 + \frac{40}{3}T_5 - \frac{7}{3}T_6 = 47 \quad \text{--- (9)}$$

Combining eqt (6-9).

$$\frac{2}{3}T_3 + 2T_4 + \frac{10}{3}T_5 - \frac{4}{3}T_6 = 11 \quad \text{--- (6)}$$

$$-4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \quad \text{--- (7)}$$

$$-\frac{2}{3}T_3 + 6T_4 + \frac{20}{3}T_5 + \frac{2}{3}T_6 = 36 \quad \text{--- (8)}$$

$$-\frac{22}{3}T_3 + 10T_4 + \frac{40}{3}T_5 - \frac{7}{3}T_6 = 47 \quad \text{--- (9)}$$

Forward elimination (eliminating T_3).

row 1 \rightarrow

row 2 \rightarrow

row 3 \rightarrow

row 4 \rightarrow

Multiply row 1 by $\frac{13}{2}$

$$\frac{13}{2} [-\frac{2}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ 11] = [-\frac{4}{3} \ \frac{26}{3} \ \frac{52}{3} \ -\frac{16}{3} \ \frac{143}{3}]$$

Eq 2 to be subtracted from this.

$$[-\frac{4}{3} \ \frac{26}{3} \ \frac{52}{3} \ -\frac{16}{3} \ \frac{143}{3}] - [-4 \ 6 \ 8 \ -7 \ 35]$$

$$\text{new row 1 } [0 \ -\frac{18}{3} \ -\frac{4}{3} \ \frac{33}{3} \ -\frac{113}{3}]$$

$$\Rightarrow -\frac{18}{3}T_4 - \frac{4}{3}T_5 + \frac{33}{3}T_6 = -\frac{113}{3} \quad \text{--- (10)}$$

Multiply row 1 by $\frac{3}{2}$

$$\frac{3}{2} [-\frac{2}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ 11] = [-1 \ 3 \ \frac{13}{2} \ -2 \ \frac{11}{2}]$$

Subtract row 3 from above;

$$[-1 \ 3 \ \frac{13}{2} \ -2 \ \frac{11}{2}] - [-\frac{2}{3} \ 6 \ \frac{20}{3} \ -\frac{2}{3} \ 36]$$

$$= [0 \ -\frac{38}{6} \ -\frac{38}{6} \ \frac{2}{2} \ -\frac{230}{6}] \text{ (new row 2)}$$

$$= -\frac{38}{6}T_4 - \frac{38}{6}T_5 + \frac{2}{2}T_6 = -\frac{230}{6} \quad \text{--- (11)}$$

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16/ENAGG/001

Mechanical Engineering

ENAG 382.

multiply row 1 by $22/7$

$$\begin{aligned} \frac{22}{7} \times \left[\begin{array}{ccc|c} -7/3 & 2 & 13/3 & -4/3 \\ 22/3 & 44/7 & 286/21 & -88/21 \\ -22/3 & 44/7 & 286/21 & -88/21 \end{array} \right] &= \left[\begin{array}{ccc|c} -22/3 & 44/7 & 286/21 & -88/21 \\ 22/3 & 44/7 & 286/21 & -88/21 \\ -22/3 & 44/7 & 286/21 & -88/21 \end{array} \right] \\ &= \left[\begin{array}{ccc|c} 0 & -26/7 & 2/7 & -13/7 \\ 22/3 & 44/7 & 286/21 & -88/21 \\ -22/3 & 44/7 & 286/21 & -88/21 \end{array} \right] \text{ (new row 3)} \\ &= -26/7 T_4 + 2/7 T_5 - 13/7 T_6 = -87/7 \quad \text{--- (12)} \end{aligned}$$

Combining eqt (10 - 12)

$$-18/7 T_4 - 4/7 T_5 + 33/7 T_6 = -113/7 \quad \text{--- (10)}$$

$$-38/7 T_4 - 380/7 T_5 + 2/7 T_6 = -230/7 \quad \text{--- (11)}$$

$$-26/7 T_4 + 2/7 T_5 - 13/7 T_6 = -87/7 \quad \text{--- (12)}$$

Forward elimination (eliminating T_4)

$$\left[\begin{array}{ccc|c} -18/7 & -4/7 & 33/7 & -113/7 \\ -38/7 & -380/7 & 2/7 & -230/7 \\ -26/7 & 2/7 & -13/7 & -87/7 \end{array} \right] \begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{array}$$

multiply row 1 by $19/9$

$$\frac{19}{9} \times \left[\begin{array}{ccc|c} -18/7 & -4/7 & 33/7 & -113/7 \end{array} \right] = \left[\begin{array}{ccc|c} -36/7 & -76/63 & 209/21 & -2147/81 \end{array} \right]$$

Subtract row 2 from the above.

$$\begin{aligned} \left[\begin{array}{ccc|c} -36/7 & -76/63 & 209/21 & -2147/81 \end{array} \right] &- \left[\begin{array}{ccc|c} -38/7 & -380/7 & 2/7 & -230/7 \end{array} \right] \\ &= \left[\begin{array}{ccc|c} 0 & 38/9 & 29/3 & -11/9 \end{array} \right] \\ &= 38/9 T_5 + 29/3 T_6 = -11/9 \quad \text{--- (13)} \end{aligned}$$

multiply row 1 by $13/9$

$$\frac{13}{9} \times \left[\begin{array}{ccc|c} -18/7 & -4/7 & 33/7 & -113/7 \end{array} \right] = \left[\begin{array}{ccc|c} -26/7 & -52/63 & 143/21 & -1469/63 \end{array} \right]$$

Subtract row 3 from the above:

$$\left[-\frac{26}{7} \quad -5\frac{2}{63} \quad 14\frac{3}{21} \mid -\frac{469}{63} \right] - \left[\frac{26}{7} \quad \frac{2}{7} \quad -\frac{13}{7} \mid -\frac{8}{7} \right]$$

$$= \left[0 \quad -\frac{10}{9} \quad +\frac{26}{3} \mid -\frac{98}{9} \right]$$

$$\therefore -\frac{10}{9}T_5 + \frac{26}{3}T_6 = -\frac{98}{9} \quad \text{--- (14)}$$

Combining eqs (13 and 14)

$$38\frac{4}{9}T_5 + \frac{29}{3}T_6 = -\frac{11}{9} \quad \text{--- (13)}$$

$$-\frac{10}{9}T_5 + \frac{26}{3}T_6 = -\frac{98}{9} \quad \text{--- (14)}$$

From eq (13) make T_6 subject of formula

$$T_6 = \left(\frac{-11}{9} - \frac{38}{9}T_5 \right) \times \frac{3}{29} \quad \text{--- (15)}$$

Backward Substitution.

Substitute T_6 in eq (14)

$$-\frac{10}{9}T_5 + \frac{26}{3} \left(\frac{-11}{9} - \frac{38}{9}T_5 \right) \frac{3}{29} = -\frac{98}{9}$$

$$-\frac{10}{9}T_5 \left(\frac{-286}{27} - \frac{988}{27}T_5 \right) \frac{3}{29} = -\frac{98}{9}$$

$$-\frac{10}{9}T_5 - \frac{286}{27} - \frac{988}{27}T_5 = -\frac{98}{9}$$

$$-\frac{142}{27}T_5 = -\frac{284}{27}$$

$$T_5 = -\frac{284}{142} = 2$$

Substitute T_5 in eq (15); $T_6 = \left(\frac{-11}{9} - \frac{38 \times 2}{9} \right) \times \frac{3}{29}$

$$\left(\frac{-11}{9} - \frac{76}{9} \right) \times \frac{3}{29}$$

$$T_6 = -1$$

Substitute T_5 and T_6 in eq (10)

$$-\frac{18}{7}T_4 = \frac{8}{7} - \frac{33}{7} = -\frac{113}{7}$$

$$-\frac{18}{7}T_4 = -\frac{113}{7} + \frac{33}{7} + \frac{8}{7}$$

$$-\frac{18}{7}T_4 = -\frac{72}{7}$$

$$T_4 = \frac{-72}{7} \times -\frac{7}{18}$$

$$T_4 = 4$$

Backward Substitution.

Substitute T_4, T_5 and T_6 in eqt (7)

from eqt (7)

$$-4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \quad \text{--- (7)}$$

$$\text{where; } T_4 = 4$$

$$T_5 = 2$$

$$T_6 = -1$$

$$-4T_3 + 6(4) + 8(2) - 7(-1) = 35$$

$$-4T_3 + 24 + 16 + 7 = 35$$

$$-4T_3 = 35 - 47$$

$$-4T_3 = -12$$

$$T_3 = 3$$

from eqt (1)

$$3T_2 - 5T_3 + 5T_5 + T_6 = -12 \quad \text{--- (1)}$$

Substitute T_3, T_5 and T_6 in eqt (1)

$$3T_2 - 5(3) + 5(2) - 1 = -12$$

$$3T_2 = -12 + 15 - 10 + 1$$

$$3T_2 = -6$$

$$T_2 = -2$$

from the initial equations.

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

Substitute T_2, T_3, T_4, T_5 and T_6

$$T_1 + (-2) - 2(3) + 4 + 3(2) - (-1) = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 = 4 + 2 + 6 - 4 - 6 - 1$$

$$T_1 = 1$$

$$T_6 = -1$$

$$T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$