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MECHANICAL ENGINEERING
ENG 382: ENGINEERING MATHEMATICS II

QUESTION

A plate (flat) of mass 'm' falling freely in air with velocity 'v' is subjected to a downward gravitational force and an upward frictional drag force due to air. If the drag force, F_D, is given by eqn (1)

$$F_D = \frac{0.3v^2}{500 + (\ln v)^3} - 0.02v \qquad — (1)$$

and the ~~inte.~~ terminal velocity is reached when the drag force equals the gravitational force

i.e $F_D = mg$ — (2)

taking the values of m and g to be 3.5kg and $9.8 m/s^2$ respectively, using a guess value of $V_1 = 0.5 m/s$ & employing fixed-point iteration method, develop a MATLAB program to estimate the terminal velocity. Take the absolute percentage relat error tolerance to be less than or equal to 1E-11.

Solution

$$F_D = \frac{0.3v^2}{500 + (\ln v)^3} - 0.02v \quad — ①$$

$$F_D = mg = 3.5 \times 9.8 = 34.3 \quad — ②$$

equating eqns ① & ②

$$mg = 34.3 = \frac{0.3v^2}{500 + (\ln v)^3} - 0.02v$$

making v^2 subject of formula

$$34.3 + 0.02v = \frac{0.3v^2}{500 + (\ln v)^3}$$

$$0.3v^2 = (34.4 + 0.02v) * (500 + (\ln v)^3)$$

$$v^2 = ((34.4 + 0.02v) + (500 + (\ln v)^2))/0.3$$

$$v = \sqrt{((34.4 + 0.02v) + (500 + (\ln v)^3))/0.3}$$

from the question;

Initial guess value $\Rightarrow v_0 = 0.5\,m/s$

Absolute % relative error, $E_a(i+1) \leq 1E-11$

MATLAB PROGRAM CODE

```
commandwindow
clear
clc
close all
format short g
syms V
V = 0.5
for i = 1 : inf
    iter(i+1) = i;
    V(i+1) = (((34.3+(0.02 *V(i)))* (500+(log(V(i)))*3
                                  ))/0.3)^0.5;
    Ea(i+1) = abs((V(i+1)-(V(i))/V(i+1))*100;
    if Ea(i+1) <= 1E-11
                break
        end
end
table = [iter' V' Ea']
```

The estimated terminal velocity is $\boxed{304.07 \text{ m/s}}$

substituting in eqn (1) ; $F_D = 34.3005 \simeq mg = 34.3$

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ASSIGNMENT II

If the maximum percentage absolute error is desired to 10.9, using the Newton-raphson Iteration method and initial guess value of e/L 0.5. Find the root of the function in the given equation (II)

(i) Manually
(ii) With the aid of MATLAB

$$f(x) = e^{-0.5x}(4-x)-2$$

N.B For the manual solution, use all the values given by the calculator

Solution.

A

$f(x) = e^{-0.5x}(4-x)-2$

$f'(x)$.

let $u = e^{-0.5x}$, $v = (4-x)$

$\delta u = -0.5e^{-0.5x}$, $\delta v = -1$

$f(x) = u\delta v + v\delta u$

$= -e^{-0.5x} - 0.5e^{-0.5x}(4-x)$

$x_0 = 0.5$ (Initial guess)

General Newton Raphson's formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(x_0) = f_{(0.5)} = 0.7258027407$

$f'(x_0') = f'_{(0.5)} = -2.141702153$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0')} = 0.8388906061$ (Root 1)

$f(x_1) = 0.07814929779$

$f'(x_1) = -1.696486032$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \cancel{0.8}\ 0.8849560003 \text{ (root 2)}$$

$f(x_2) = 1.236575203 \times 10^{-3}$

$f'(x_2) = -1.643060762$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.885708605 \text{ (root 3)}$$

$f(x_3) = 3.23583557 \times 10^{-3}$

$f'(x_3) = -1.642200929$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.885708802 \text{ (root 4)}$$

$f(x_4) = 7.845 \times 10^{-12}$

$f'(x_4) = -1.642200704$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.885708802 \text{ (root 5)}$$

$\therefore$ 0.885708802 is the root of eqn (1.1).

MATLAB PROGRAM CODE

```
function [x1, err, relerr] = assign1(x0, max1, tol, iter, f, fprime)
x0 = 0.5 ;
max1 = 100 ;
tol.0.0000000001
iter = 1
f = @ (x) ((exp(0.5*x))*(4-x))-2 ;
fprime = @ (x) (-exp(-0.5*x)) + ((-0.5*exp(-0.5*x))*(4-x));

for i = 1 : max 1
x1 = x0 - feval(f, x0 /feval(fprime, x0);
err = abs(x1-x0); relerr = abs(x1-x0)/x1 ;
fprintf(%2 of %10 of 10-10f %10.10f %10.10f \x; Ser x0, x
err, relerr)
x0 = x1 ; iter = 1 + iter ;
if err < = tol, break , end
end
```