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16/ENG04/045
ELECT/ELECT
ENA 382

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ASSIGNMENT 2

A) Manually

$$f(x) = e^{-0.5x} (4-x) - 2$$

Given initial guess value of $0.5 = x_0$

Maximum percentage absolute error = 10^{-9}

1) Find the root of the function

Solution

$$f(x) = (4-x)e^{-0.5x} - 2$$

To get the root:

$$\text{when } x=0, f(x) = (4-0)e^{-0.5(0)} - 2 = 2$$

$$f(0) = 2$$

$$\text{when } x=1, f(x) = (4-1)e^{-0.5(1)} - 2 = -0.180$$

for $f(x)$

$$\begin{aligned} \text{expanding } f(x) &= e^{-0.5x} (4-x) - 2 \\ &= 4e^{-0.5x} - xe^{-0.5x} - 2 \end{aligned}$$

$$f'(x) \Rightarrow \frac{dy}{dx} (f(x))$$

$$f'(x) = \frac{d}{dx} [e^{-0.5x} (4-x)] - \frac{d}{dx} (2)$$

$$= e^{-0.5x} \frac{d}{dx} (4-x) + (4-x) \frac{d}{dx} (e^{-0.5x}) - 0$$

$$= e^{-0.5x} (-1) + (4-x) (-0.5e^{-0.5x}) - 0$$

$$= -e^{-0.5x} + (4-x) (-0.5e^{-0.5x})$$

$$= -e^{-0.5x} - 0.5e^{-0.5x} (4-x)$$

$$f'(x) = -0.5xe^{-0.5x} - 3e^{-0.5x}$$

$$f'(x) = e^{-0.5x} [-0.5x - 3]$$

using Newton-Raphson Method,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\text{for percentage absolute error} = \left[\frac{x_{k+1} - x_k}{x_{k+1}} \right] \times 100$$

for Iter 1;

$$\text{let } x_k = x_0 = 0.5$$

$$f(x_0) = (4 - 0.5)e^{-0.5(0.5)} - 2$$

$$f(x_0) = 0.7258027407$$

$$f'(x_0) = e^{-0.5(0.5)} [(0.5 \times 0.5) - 3]$$

$$= -2.141702158 = -2.141702153$$

$$x_{k+1} = 0.5 - \frac{0.7258027407}{-2.141702153}$$

$$= 0.838890606$$

$$x_{k+1} = 0.838890606$$

$$\% \text{ absolute error} = 40.39747299\%$$

for Iter 2;

$$\text{let } x_k = x_1 = 0.838890606$$

$$f(x_1) = (4 - 0.838890606)e^{-0.5(0.838890606)} - 2$$

$$f(x_1) = 0.07814929794$$

$$f'(x_1) = e^{-0.5(0.838890606)} [(0.5 \times 0.838890606) - 3]$$

$$f'(x_1) = -1.696486032$$

$$\therefore x_{k+1} = 0.838890606 - \frac{0.07814929794}{-1.696486032}$$

$$x_{k+1} = 0.8849560003$$

Table of Results

i	x_k	$f(x_k)$	$f'(x_k)$	x_{k+1}	% abs. error
1	0.5	0.7258027407	-2.141702153	0.838890606	40.39747299
2	0.838890606	0.07814929794	-1.696486032	0.8849560003	5.205388094
3	0.8849560003	0.00123657519	-1.643060762	0.885708605	0.08497203912
4	0.885708605	3.2352141×10^{-7}	-1.642200929	0.885708802	$2.224261137 \times 10^{-5}$
5	0.885708802	7.851×10^{-12}	-1.642200704	0.885708802	0