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16/ENG06/021
MECHANICAL ENGINEERING
ENG 382

ASSIGNMENT 2

1. $f(x) = e^{-0.5x}[4-x]-2$

Given initial guess value of $0.5 = x_0$, maximum percentage absolute error = 1E-9

(i) Find the root of the function manually and with the aid of matlab using Newton Raphson Iteration Method

Soln

From Newton Raphson Iteration

$$x_{i+1} = x_i - \frac{f(x)}{f'(x)}$$

$$f(x) = e^{-0.5x}[4-x]-2$$

$$f'(x) = \quad u = e^{-0.5x} \quad v = 4-x$$

$$du/dx = -e^{0.5-0.5x}, \quad dv/dx = -1$$

from product rule

$$u\,dv/dx + v\,du/dx$$

$$-e^{-0.5x} + [-0.5e^{-0.5x} \times [4 - x]]$$

$$-e^{-0.5x} - 2e^{-0.5x} + 0.5e^{-0.5x}$$

$$f'(x)\, x\, 0.5e^{-0.5x} - 3e^{-0.5x}$$

$$e^{-0.5x}[0.5x - 3]$$

$$\text{Percentage absolute error} = \left[\frac{x_{i+1} - x_i}{x_{i+1}}\right] \times \frac{100}{1}$$

→ for 1st Iteration;

Let $x_i = 0.5$

$$f(x_0) = [4-0.5]e^{-0.5[0.5]} - 2$$

$$f(x_0) = 0.7258027407$$

~~$f'(x_0) = e^{-0.5}[0.5$~~

$$f'(x_0) = e^{-0.5(0.5)}[(0.5 \times 0.5) - 3]$$

$$= -2.141702158$$

$$\simeq -2.$$

$$x_{i+1} = 0.5 - \frac{0.7258027407}{-2.141702153}$$

$$x_{i+1} = 0.838890606$$

$$\% \text{ absolute error} = \left[\frac{0.838890606 - 0.5}{0.838890606}\right] \times 100\% = 40.39747299\%$$

→ for 2nd Iteration;

$$x = 0.838890606$$

$$f(x_1) = [4 - 0.838890606] e^{-0.5[0.838890606]} - 2$$

$$f(x_1) = 0.07814929794$$

$$f'(x_1) = e^{-0.5 \times [0.838890606]} [[0.5 \times 0.838890606] - 3]$$

$$f'(x_1) = -1.696486032$$

$$x_{i+1} = 0.838890606 - \left[\frac{0.07814929794}{-1.696486037}\right]$$

$$x_{i+1} = 0.884956003$$

$$\% \text{ absolute error} = \left[\frac{0.884956003 - 0.838890606}{0.884956003}\right] \times \frac{100}{1} = 5.205388094\%$$

→ for 3rd Iteration;

$$x_i = 0.8849560003$$

$$f(x_2) = [4 - 0.8849560003] e^{-0.5[0.8849560003]}$$

$$f(x_2) = 0.00123657519$$

$$f'(x_2) = e^{[-0.5 \times 0.8849560003]} [[0.5 \times 0.8849560003] - 3]$$

$$f'(x_2) = -1.643060762$$

$$x_{i+1} = 0.8849560003 - \left[\frac{0.00123657519}{-1.643060762}\right] = 0.885708605$$

$$\% \text{ absolute error} = \left[\frac{0.885708605 - 0.8849560003}{0.885708605}\right] \times 100$$

$$\% \text{ absolute error} = 0.08497203912\%$$

→ for 4th Iteration;

$$x_i = 0.885708605$$

$$f(x_3) = [4 - 0885708605] e^{-0.5[0.885708605]} - 2$$

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$f(x_3) = 3.2352141 \times 10^{-7}$

$f'(x_3) = e^{[-0.5 \times 0.885708605]}[(0.5 \times 0.885708605) - 3]$

$f'(x_3) = -1.642200929$

$$x_{i+1} = 0.885708605 - \left[\frac{3.2352141 \times 10^{-7}}{-1.642200929}\right] = 0.885708802$$

$$\% \text{ absolute error} = \left[\frac{0.885708802 - 0.885708605}{0.885708802}\right] \times 100\% = 2.224261137 \times 10^{-5}\%$$

→ for 5th Iteration;

$x_i = 0.885708802$

$f(x_4) = [4 - 0.885708802]\, e^{-0.5[0.885708802]} - 2$

$f(x_4) = 7.851 \times 10^{-12}$

$f'(x_4) = e^{[0.5 \times 0.885708802]}[(0.5 \times 0.885708802) - 3]$

$f'(x_4) = -1.642200704$

$$x_{i+1} = 0.885708802 - \left[\frac{7.83 \times 10^{-12}}{-1.642200704}\right] = 0.885708802$$

$$\% \text{ absolute error} = \left[\frac{0.885708802 - 0.885708802}{0.885708802}\right] \times \frac{100}{1} = 0\%$$

Iteration	$x_{(i+1)}$	% error
1	0.838890606	40.39747299
2	0.849560003	5.205388094
3	0.885708605	0.08497203912
4	0.885708802	$2.224261137 \times 10^{-5}$
5	0.885708802	0