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16/ENG07/011
Petroleum Engineering
ENG 382
Assignment Two

(a) Manually

$f(x) = e^{-0.5x}(4-x) - 2$ given initial guess value of 0.5, Maximum percentage absolute error = 1E-9

Therefore the root of the function is;

Solution

$f(x) = (4-x)e^{-0.5x} - 2$

when $x = 0$

$f(x) = (4-0)e^{-0.5(0)} - 2 = 2$

$f(x) = 2$

when $x = 1$

$f(x) = (4-1)e^{-0.5(1)} - 2 = -0.18$

Therefore, to find $f'(x)$;

Expanding $f(x) = e^{-0.5x}(4-x) - 2$

$= 4e^{-0.5x} - xe^{-0.5x} - 2$ OR differentiate $f(x)$ to get $f'(x)$ using product rule

$f'(x) = \frac{d}{dx}\left[e^{-0.5x}(4-x)\right] - \frac{d}{dx}(2)$

$= e^{-0.5x}\frac{d}{dx}(4-x) + (4-x)\frac{d}{dx}(e^{-0.5x}) - 0$

$= e^{-0.5x} \cdot -1 + (4-x) \cdot -0.5e^{-0.5x}$

$= -e^{-0.5x} + (4-x) \cdot -0.5e^{-0.5x}$

$= 4 \cdot -0.5e^{-0.5x} + x \cdot 0.5e^{-0.5x} - e^{-0.5x}$

$= x \cdot 0.5e^{-0.5x} - 2e^{-0.5x} - e^{-0.5x}$

$f'(x) = 0.5e^{-0.5x} - 3e^{-0.5x}$

Using Newton-Raphson Method,

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

for Percentage absolute error $= \left[\frac{x_{k+1} - x_k}{x_{k+1}}\right] \times 100\%$

For Iter 1

let $x_k = x_0' = 0.5$

$f(x_0) = (4-0.5)e^{-0.5(0.5)} - 2$

$f(x_0) = 0.7258027407$

$f'(x_0) = e^{-0.5(0.5)}\left[(0.5\times0.5)-3\right]$

$= -2.1417$

$$x_{k+1} = 0.5 - \frac{0.7258027407}{-2.1417}$$

$x_{k+1} = 0.8388909468$

% absolute error $= \left[\frac{0.8389 - 0.5}{0.8389}\right] \times 100\%$

% absolute error $= 40.398\%$

For iter 2

let $x_k = 0.8389 = x_1$

$f(x_1) = (4-0.8389)e^{-0.5(\cancel{0.8888909468})} - 2$

$f(x_1) = 0.0781487196\,5$

$f'(x_1) = e^{-0.5\times(0.838890468)}\left[(0.5\times0.838890468)-3\right]$

$f'(x_1) = -1.696486194$

$$x_{k+1} = 0.8388909468 - \left[\frac{0.0781487965}{-1.696486194}\right]$$

$$x_{k+1} = 0.8849559958$$

$$\% \text{ absolute error} = \left[\frac{0.8849559958 - 0.8388909468}{0.8849559958}\right] \times 100\%$$

$$\% \text{ absolute error} = 0.05205349904 \times 100\%$$
$$= 5.21\%$$

for iter 3:

let $x_k = 0.8849559958 = x_2$

$$f(x_2) = (4 - 0.8849559958)\, e^{-0.5(0.8849559958)} - 2$$

$$f(x_2) = 1.236582455 \times 10^{-3}$$
$$= 0.001236582455$$

$$f'(x_2) = e^{(-0.5 \times 0.8849559958)}\left[(0.5 \times 0.8849559958) - 3\right]$$

$$f'(x_2) = -1.643060767$$

$$x_{k+1} = 0.8849559958 - \left[\frac{0.001236582455}{-1.643060767}\right]$$

$$x_{k+1} = 0.8857086067$$

$$\% \text{ absolute error} = \left[\frac{0.8857086067 - 0.8849559958}{0.8857086067}\right] \times 100\%$$

$$= 0.0849727432\,\%$$

for iter 4:

let $x_k = 0.8857086067 = x_3$

$$f(x_3) = (4 - 0.8857086067)\, e^{-0.5(0.8857086067)} - 2$$

$$f(x_3) = 3.2053706 \times 10^{-7}$$
$$= 3.2053706 \times 10^{-7}$$

$$f'(x_3) = e^{(-0.5 \times 0.8857086067)}\left[(0.5 \times 0.8857086067) - 3\right]$$

$$f'(x_3) = -1.642200927$$

$$\therefore x_{k+1} = 0.8857086067 - \left[\frac{3.20537006 \times 10^{-7}}{-1.642200987}\right]$$

$$x_{k+1} = 0.8857088019$$

$$\% \text{ absolute error} = \left[\frac{0.8857088019 - 0.8857086067}{0.8857088019}\right]$$

$$\times 10^{\circ}\% = 2.203884613 \times 10^{-5}\%$$

For iter 5,

$$x_k = 0.885708819 = x_4$$

$$f(x_4) = (4 - 0.885708819)\, e^{-0.5(0.885708819)} - 2$$

$$f(x_4) = -2.7909567 \times 10^{-8}$$

$$f'(x_4) = e^{(-0.5 \times 0.885708819)}\left[(0.5 \times 0.885708819) - 3\right]$$

$$f'(x_4) = -1.642200684$$

$$x_{k+1} = 0.885708819 - \left[\frac{-2.7909567 \times 10^{-8}}{-1.642200684}\right]$$

$$x_{k+1} = 0.885708802$$

$$\% \text{ absolute error} = \left(\frac{0.885708802 - 0.885708819}{0.885708802}\right)$$

$$\times 10^{\circ}\%$$

$$\% \text{ absolute error} = 0\%$$