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16/ENG04/047
ASSIGNMENT 2

1.)

$f(x) = e^{-0.5x}(4-x) - 2$

Finding the root

When $x = 0$; $f(x) = e^{-0.5(0)}(4-0) - 2 = 2$.

When $x = 1$; $f(x) = e^{-0.5(1)}(4-1) - 2 = -0.180408$.

$\therefore f'(x)$

Expand $f(x) = e^{-0.5x}(4-x) - 2$ OR $fx = e^{-0.5x}(4-x) - 2$

$= 4e^{-0.5x} - xe^{-0.5x} - 2$

Differentiating with Product Rule

$$f'(x) = \frac{d}{dx}\left\{e^{-0.5x}(4-x)\right\} - \frac{d}{dx}\left\{2\right\}$$

$$= e^{-0.5x}\cdot\frac{d}{dx}(4-x) + 4-x\cdot\frac{d}{dx}(e^{-0.5x}) - 0$$

$$= e^{-0.5x}\cdot(-1) + 4-x\cdot(-0.5e^{-0.5x})$$

$$= -e^{-0.5x} + (4-x)(0.5e^{-0.5x})$$

$$= -e^{-0.5x} + 4(0.5e^{-0.5x}) + x(0.5e^{-0.5x})$$

$$= x\,0.5e^{-0.5x} - 2e^{-0.5x} - e^{-0.5x}$$

$f'(x) = 0.5xe^{-0.5x} - 3e^{-0.5x}$

$f'x = e^{-0.5x}\,0.5x - 3$

Using Newton-Raphson Method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

% Absolute Error, $\left\{\frac{x_{k+1} - x_k}{x_{k+1}}\right\} \times 100\%$

For iter 1;

Let $x_k = 0.5 = x_0$

$f(x_0) = (4-0.5)e^{-0.5(0.5)} - 2$

$f(x_0) = 0.7258$

$f'(x) = e^{-0.6(0.5)}[(0.5 \times 0.5) - 3]$

$= -2.142$

$x_{k+1} = 0.5 - \frac{0.7258}{-2.142}$

$= 0.8388$

$\%\text{error} = \left(\frac{0.8388 - 0.5}{0.8388}\right) \times 100$

$\%\text{error} = 40.396$

For iter 2;

Let $x_k = 0.8388 = x_1$

$f(x_1) = (4 - 0.8388)e^{-0.5(0.8388)} - 2$

$f(x_1) = 0.0783$

$f'(x_1) = e^{-0.5 \times 0.8388}[(0.5 \times 0.8388) - 3]$

$f'(x_1) = -1.6966$

$\therefore x_{k+1} = 0.8388 - \frac{0.0783}{-1.6966}$

$x_{k+1} = 0.88496$

$\%\text{error} = \left(\frac{0.88496 - 0.8388}{0.88496}\right) \times 100$

$= 5.2151$

For iter 3;

Let $x_k = 0.88496 = x_2$

$f(x_2) = (4 - 0.88496)e^{-0.5(0.88496)} - 2$

$f x_2 = 0.00123$

$f'(x_2) = e^{-0.5 \times 0.88496}[(0.5 \times 0.88496) - 3]$

$f'(x_2) = -1.64306$

$x_{k+1} = 0.88496 - \frac{0.00123}{-1.64306} = 0.885708605$

$\%error = \left(\frac{0.88571 - 0.88496}{0.88571}\right) \times 100$

$= 0.0846$

For iter 4;

Let $x_k = 0.88571$

$f(x_3) = (4 - 0.88571)e^{-0.5(0.88571)} - 2$

$f(x_3) = -1.96734 \times 10^{-6}$

$f'(x_i) = e^{-0.5 \times 0.88571}((0.5 \times 0.88571) - 3)$

$f'(x_i) = -1.64220$

$x_{(k+1)} = 0.8857 - \frac{1.96734 \times 10^{-6}}{-1.64220} = 0.88571$

$\%error = \left(\frac{0.88571 - 0.88571}{0.88571}\right) \times 100$

$= 0$

i	x_k	$f x_k$	$f' x_k$	x_{k+1}	%abs error
0	0.5	0.7258	-2.142	0.8388	40.396
1	0.8388	0.0783	-1.6966	0.88496	5.2151
2	0.88496	0.00123	-1.64306	0.88571	0.0846
3	0.88571	-1.9674×10^{-6}	-1.64220	0.88571	0

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