

$$f(x) = e^{-0.5x} (4-x) - 2$$

Given initial guess value of  $0.5 = x_0$

Maximum percentage absolute error =  $10^{-9}$

is find the root of the function.

Solution.

$$f(x) = (4-x)e^{-0.5x} - 2$$

To find the root

$$\text{When } x=0, f(x) = (4-0)e^{-0.5} - 2 = 2$$

$$f(x) = 2$$

$$\text{When } x=1, f(x) = (4-1)e^{-0.5(1)} - 2 = 0.180408$$

$$f(x) = -0.180$$

Therefore, to find  $f'(x)$ .

$$\text{Expanding } f(x) = e^{-0.5x} - xe^{-0.5x} - 2$$

or

$$f(x) = e^{-0.5x} (4-x) - 2$$

$f(x) \Rightarrow$  differentiating using product rule

$$f(x) = \frac{d}{dx} \int e^{-0.5x} (4-x) \cdot \frac{d}{dx} [2]$$

$$= e^{-0.5x} \cdot \frac{d}{dx} \int e^{-0.5x} (4-x)$$

$$= e^{-0.5x} \cdot \frac{d}{dx} (4-x) + (4-x) \cdot \frac{d}{dx} (e^{-0.5x}) = 0$$

$$= e^{-0.5x} - 1 + (4-x) \cdot \frac{d}{dx} (e^{-0.5x}) = 0$$

$$= -e^{-0.5x} - 1 + (4-x) \cdot 0.5e^{-0.5x}$$

$$= 4 \cdot -0.5e^{-0.5x} + x \cdot 0.5e^{-0.5x} - e^{-0.5x}$$

$$= 2x \cdot 0.5e^{-0.5x} - 2e^{-0.5x} - e^{-0.5x}$$

$$f'(x) = 0.5xe^{-0.5x} - 3e^{-0.5x}$$



$$f'(x) = e^{-0.5x} [0.5x - 3]$$

using Newton Raphson Method.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\text{For percentage absolute error} = \left[ \frac{x_{k+1} - x_k}{x_{k+1}} \right] \times 100\%$$

For iter 1;

$$\text{Let } x_k = 0.5 = x_0$$

$$f(x_0) = 4 - 0.5)e^{-0.5(0.5)} - 2$$

$$= 0.7258027407$$

$$f'(x_0) = e^{-0.5(0.5)} [0.5 \times 0.5 - 3]$$

$$= 2.141702153 = -2.141702153$$

$$\therefore x_{k+1} = 0.5 - \frac{0.7258027407}{-2.141702153}$$

$$= 0.838890606$$

$$x_{k+1} = 0.838890606$$

$$\% \text{ absolute error} = \left[ \frac{0.838890606 - 0.5}{0.838890606} \right] \times 100\%$$

$$\% \text{ absolute error} = 40.37747299\%$$

Iter 2:

$$\text{Let } x_k = 0.838890606 = x_1$$

$$f(x_1) = (4 - 0.838890606)e^{-0.5(0.838890606)} - 2$$

$$f(x_1) = 0.07814929794$$

$$f'(x_1) = e^{-0.5 \times 0.838890606} [0.5 \times 0.838890606 - 3]$$

$$f'(x_1) = -1.696986032$$

$$\therefore x_{k+1} = 0.838890606 - \left[ \frac{0.07814929794}{-1.696986032} \right]$$

$$x_{k+1} = 0.8849560003$$

Summary of obtained Results.  
Table.

$i$	$x_k$	$f(x_k)$	$f'(x_k)$	$x_{k+1}$	$2. \text{ abs. err}$
1	0.5	0.7258027907	-2.141702153	0.888890606	40.3774729
2	0.8849560003	0.07814929794	-1.696486032	0.8849560003	5.20538809
3	0.885708605	0.00123657519	-1.643060762	0.885708605	0.084972089
4	$3.23521411 \times 10^{-7}$	$3.23521411 \times 10^{-7}$	-1.042200929	0.885708602	2.224261137x
5	$7.851 \times 10^{-12}$	$7.851 \times 10^{-12}$	-1.042200704	0.885708602	0