

Assignment III

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ENG 382:

Answer

The augmented matrix is

$$\tilde{A}^{(0)} = \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & 1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

$$\begin{array}{l} \tilde{A}^{(0)} = \\ \frac{E_2}{2} - E_1 \Rightarrow \\ \frac{E_3}{2} - E_1 \Rightarrow \\ \frac{E_4}{5} - E_1 \Rightarrow \\ \frac{E_5}{-3} - E_1 \Rightarrow \\ \frac{E_6}{4} - E_1 \Rightarrow \end{array} \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -1.5 & 2.5 & 0 & -2.5 & -0.5 & 6 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -0.6 & 1.8 & -1.2 & -2.4 & 1.2 & -4.6 \\ 0 & -0.667 & 1.333 & -2 & -3.333 & 0 & -9.333 \\ 0 & -0.25 & 2.25 & -2.5 & -3.75 & 0.5 & -16.75 \end{array} \right]$$

Divide through E_3 by 2 and Swap (Pivot) with E_2

$$\tilde{A}^{(0)} = \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & -1.5 & 2.5 & 0 & -2.5 & -0.5 & 6 \\ 0 & -0.6 & 1.8 & -1.2 & -2.4 & 1.2 & -4.6 \\ 0 & -0.667 & 1.333 & -2 & -3.333 & 0 & -9.333 \\ 0 & -0.25 & 2.25 & -2.5 & -3.75 & 0.5 & -16.75 \end{array} \right]$$

$$A^{(3)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1.167 & -1 & -2.167 & 0.667 & -3.5 \\ 0 & 0 & 2.3 & -3 & -4.833 & 3 & -17.167 \\ 0 & 0 & 1.499 & -3.999 & -3.497 & 1 & -23.493 \\ 0 & 0 & 8.3 & -11 & -13.5 & 3 & -32.5 \end{bmatrix}$$

$\frac{E_3}{1.167} + E_2 \Rightarrow$
 $\frac{E_4}{0.6} + E_1 \Rightarrow$
 $\frac{E_5}{0.667} + E_2 \Rightarrow$
 $\frac{E_6}{0.23} + E_2 \Rightarrow$

Divide through E_4 by 2.3 and Swap with E_3

$$A^{(4)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 1.167 & -1 & -2.167 & 0.667 & -3.5 \\ 0 & 0 & 1.499 & -3.999 & -3.497 & 1 & -23.493 \\ 0 & 0 & 8.3 & -11 & -13.5 & 3 & -32.5 \end{bmatrix}$$

$$A^{(5)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 0.343 & -0.76 & -0.628 & 2.154 \\ 0 & 0 & 0 & -1.468 & -1.734 & -0.333 & -8.803 \\ 0 & 0 & 0 & -0.094 & 0.109 & -0.847 & 0.691 \end{bmatrix}$$

$\frac{E_4}{1.167} - E_3 \Rightarrow$
 $\frac{E_5}{0.499} - E_3 \Rightarrow$
 $\frac{E_6}{8.3} - E_3 \Rightarrow$

Divide through E_4 by 0.343

$$A^{(6)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & -1.468 & -1.734 & -0.333 & -8.803 \\ 0 & 0 & 0 & -0.094 & 0.109 & -0.847 & 0.691 \end{bmatrix}$$

$$A^{(5)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 0.959 & 2.194 & -0.252 \\ 0 & 0 & 0 & 0 & -1.382 & 10.842 & -13.634 \end{bmatrix}$$

$\frac{E_5}{-1.468} - E_4$
 $\frac{E_6}{-0.094} - E_4$

Divide through E_5 by 0.959

$$A^{(6)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 1 & 2.288 & -0.294 \\ 0 & 0 & 0 & 0 & -1.382 & 10.842 & -13.631 \end{bmatrix}$$

$$A^{(6)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 1 & 2.288 & -0.294 \\ 0 & 0 & 0 & 0 & 0 & -10.133 & 10.157 \end{bmatrix}$$

$\frac{E_4}{-1.382} - E_5$

Backward Substitution

$A^{(6)}$ From $E_6: -10.133 T_6 = 10.157$

$T_6 = -1.002 \approx -1$

From $E_5: T_5 + 2.288 T_6 = -0.294$

$T_5 = 1.997 \approx 2$

$E_4: T_4 + 0.222 T_5 - 1.831 T_6 = 6.280$

$T_4 = -0.222(1.997) + 1.831(-1.002) + 6.280$

$T_4 = 4.002 \approx 4$

From E_3 : $\bar{T}_3 - 1.2\bar{T}_4 - 1.933\bar{T}_5 + 1.2\bar{T}_6 = -6.867$

$$\bar{T}_3 \approx +1.2(4.002) + 1.933(1.999) - 1.2(-1.002)$$

$$6.867$$

$$\bar{T}_3 = 3.002 \approx 3$$

From E_2 : $\bar{T}_2 - 0.5\bar{T}_3 - \bar{T}_4 - 0.5\bar{T}_5 + \bar{T}_6 = -9.5$

$$\bar{T}_2 = 0.5(3.002) + 4.002 + 0.5(1.999) - (-1.0)$$

$$\bar{T}_2 = -1.996 \approx -2$$