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Assignment (2)

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Level: 300L

Question (1)

(1) $f(x) = e^{-0.5x}(4-x) - 2$

Given Initial guess value of $0.5 = x_0$

Maximum percentage absolute error $= 1E^{-9}$

(i) find the root of the function

Solution

$f(x) = (4-x)e^{-0.5x} - 2$

To find the root

when $x = 0$ $f(x) = (4-0)e^{-0.5(0)} - 2 = 2$

when $x = 1$, $f(x) = (4-1)e^{-0.5(1)} - 2 = -0.180408$

To find $f'(x)$; $f(x) = e^{-0.5x}(4-x) - 2$
$= 4e^{-0.5x} - xe^{-0.5x} - 2$

$f'(x) \Rightarrow$ Using Product rule

$f'(x) \Rightarrow \frac{d}{dx}\left[e^{-0.5x}(4-x)\right] - \frac{d}{dx}[2]$

$= e^{-0.5x}\frac{d}{dx}(4-x) + (4-x)\cdot\frac{d}{dx}(e^{-0.5x})$

$= e^{-0.5x}\cdot -1 + (4-x)\cdot -0.5e^{-0.5x}$

$= -e^{-0.5x} + (4-x) \cdot -0.5e^{-0.5x}$

$= 4(-0.5e^{-0.5x}) + x(0.5e^{-0.5x}) - e^{-0.5x}$

$= x \cdot 0.5e^{-0.5x} - 2e^{-0.5x} - e^{-0.5x}$

$f'(x) = 0.5xe^{-0.5x} - 3e^{-0.5x}$

$f'(x) = e^{-0.5x}[0.5x - 3]$

Using Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Recall:

Percentage absolute error $= \left[\frac{x_{n+1} - x_n}{x_{n+1}}\right] \times 100\%$

For the first iteration;

Let $x_n = 0.5 = x_0$

$f(x_0) = (4-0.5)e^{-0.5(0.5)} - 2$

$f(x_0) = 0.7258027407$

$f'(x_0) = e^{-0.5(0.5)}[(0.5 \times 0.5) - 3]$

$= -2.141702158$.

$\therefore x_{n+1} = 0.5 - \frac{0.7258027407}{-2.141702153}$

% Absolute Error $= \left[\frac{0.888890606 - 0.5}{0.888890606}\right] \times 100\%$

$= 40.39747299\%$

for the second iteration

Let $x_n = 0.838890606 = x_1$

$f(x_1) = (7 - 0.838890606)\, e^{-0.5(0.838890606)} - 2$

$f(x_1) = 0.0781492979 4$

$f'(x_1) = e^{-0.5(0.838890606)} \left[(0.5 \times 0.838890606) - 3 \right]$

$f'(x_1) = -1.696486032$

$\therefore\ x_{n+1} = 0.838890606 - \left[\dfrac{0.07814929794}{-1.696486032} \right]$

$x_{n+1} = 0.8849560003$

$\%$ absolute error $= \left[\dfrac{0.8849560003 - 0.838890606}{0.8849560003} \right] \times 100\%$

$= 5.205388094\ \%$

for the 3rd iteration;

Let $x_n = 0.8849560003 = x_2$

$f(x_2) = (7 - 0.8849560003)\, e^{-0.5(0.8849560003)} - 2$

$f(x_2) = 0.00123657519$

$f'(x_2) = e^{(-0.5 \times 0.8849560003)} \left[(0.5 \times 0.8849560003) - 3 \right]$

$f'(x_2) = -1.643060762$

$\therefore\ x_{n+1} = 0.8849560003 - \left[\dfrac{0.00123657519}{-1.643060762} \right]$

(4)

$X_{n+1} = 0.885708605$

$$\% \text{ Absolute error} = \left[\frac{0.885708605 - 0.884956 0 0 03}{0.885708605}\right] \times 100\%$$

$$= 0.084972 0 3912\%$$

for the fourth iteration 4;

Let $x_n = 0.885708605 = x_3$

$$f(x_3) = (4 - 0.885708605)e^{-0.5(0.885708605)} - 2$$

$$f(x_3) = 3.23521411 \times 10^{-7}$$

$$f'(x_3) = e^{(-0.5 \times 0.885708605)}[(0.5 \times 0.885708605) - 3]$$

$$f'(x_3) = -1.642200929$$

$$\therefore x_{n+1} = 0.885708605 - \left[\frac{3.23521411 \times 10^{-7}}{-1.642200929}\right]$$

$$x_{n+1} = 0.885708802$$

$$\% \text{ Absolut error} = \left[\frac{0.885708802 - 0.885708605}{0.885708802}\right] \times 100\%$$

$$= 2.224261137 \times 10^{-5}\%$$

for the fifth iteration

$x_n = 0.885708802 = x_4$

$$f(x_4) = (4 - 0.885708802)e^{-0.5(0.885708802)} - 2$$

$$f(x_4) = 7.851 \times 10^{-12}$$

$$f'(x_4) = e^{(-0.5 \times 0.885708802)}[(0.5 \times 0.885708802) - 3]$$

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Continuation of the fifth Iteration

$f'(x_4) = -1.642200704$

$$x_{n+1} = 0.885708802 - \left[\frac{7.851 \times 10^{-12}}{-1.642200704}\right]$$

$$x_{k+1} = 0.885708802$$

$$\% \text{ Absolute error} = \left[\frac{0.885708802 - 0.885708802}{0.885708802}\right] \times 100\%$$

$$\% \text{ Absolute error} = 0\%$$