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 Matric: 161ENG071023
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 Course: ENG 382

Assignment (3)

Gaussian Elimination method
 Manually

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \quad \text{--- (1)}$$

$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \quad \text{--- (2)}$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \quad \text{--- (3)}$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \quad \text{--- (4)}$$

$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \quad \text{--- (5)}$$

$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27 \quad \text{--- (6)}$$

The Corresponding Matrix is

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -2 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

Augmenting the matrix, we have

$$\begin{array}{c|cccccc|c} & A & B & C & D & E & F & \\ \hline 1 & 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 3 & 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 4 & 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ 5 & -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 6 & 4 & 3 & 1 & -6 & -2 & -2 & -27 \end{array}$$

Making $A_{1,1} = 0$

Using formula Row 2 - $\left(\frac{A_{2,1}}{A_{1,1}}\right) \times \text{Row 1}$

$$A_{2,1}: 2 - (2)1 = 0$$

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Pivot equation $T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$

making $A_{2,1}$ the Pivot

$$\times_2 [T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4]$$

$$= 2T_1 + 2T_2 - 4T_3 + 2T_4 + 6T_5 - 2T_6 = 8 \quad \dots \quad (2')$$

$$\times_1 [T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4]$$

$$= T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \quad \dots \quad (3')$$

$$\times_5 [T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4] \quad \dots \quad (4')$$

$$= 5T_1 + 5T_2 - 10T_3 + 5T_4 + 15T_5 - 5T_6 = 20 \quad \dots \quad (4')$$

$$-3 \times_1 [T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4]$$

$$= -3T_1 - 3T_2 + 6T_3 - 3T_4 - 9T_5 + 3T_6 = -12 \quad \dots \quad (5')$$

$$+4 \times_1 [T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4]$$

$$= 4T_1 + 4T_2 - 8T_3 + 4T_4 + 12T_5 - 4T_6 = 16 \quad \dots \quad (6')$$

Subtracting $2'$, $3'$, $4'$, $5'$ and $6'$ from their Original equation
 $2, 3, 4, 5$ and 6
 for eqn $(2-2')$

$$\begin{aligned} & 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ & - (2T_1 + 2T_2 - 4T_3 + 2T_4 + 6T_5 - 2T_6 = 8) \\ & = 0T_1 - 3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 = 12 \quad \dots \quad (2'') \end{aligned}$$

for eqn $(3-3')$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \quad \dots \quad (T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4)$$

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$$= 0T_1 + 2T_2 - 1T_3 - 2T_4 - T_5 + 2T_6 = -19 \dots (3'')$$

for eqn (4-4')

$$\begin{aligned} 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 &= -3 \\ -(5T_1 + 5T_2 - 10T_3 + 5T_4 + 15T_5 - 5T_6 &= 20) \\ = 0T_1 - 3T_2 + 9T_3 - 6T_4 - 13T_5 + 6T_6 &= -23 \dots (4'') \end{aligned}$$

for eqn (5-5')

$$\begin{aligned} -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 &= 16 \\ -(-3T_1 - 3T_2 + 6T_3 - 3T_4 - 9T_5 + 3T_6 &= -12) \\ = 0T_1 + 2T_2 + (-4T_3) + 6T_4 + 10T_5 + 0T_6 &= 28 \dots (5'') \end{aligned}$$

for eqn (6-6')

$$\begin{aligned} 4T_1 + 3T_2 + T_3 &= 6T_4 - 3T_5 - 2T_6 = -27 \\ -(4T_1 + 4T_2 - 8T_3 + 4T_4 + 12T_5 - 4T_6 &= 16) \\ = 0T_1 - 1T_2 + 9T_3 - 10T_4 - 15T_5 + 2T_6 &= -43 \end{aligned}$$

Rewriting the equations we have

$$\begin{aligned} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 &= 4 \dots (1'') \\ 0T_1 - 3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 &= 12 \dots (2'') \\ 0T_1 + 2T_2 - 1T_3 - 2T_4 - T_5 + 2T_6 &= -19 \dots (3'') \\ 0T_1 - 3T_2 + 9T_3 - 6T_4 - 13T_5 + 6T_6 &= -23 \dots (4'') \\ 0T_1 + 2T_2 - 4T_3 + 6T_4 + 10T_5 + 0T_6 &= 28 \dots (5'') \\ 0T_1 - T_2 + 9T_3 - 10T_4 - 15T_5 + 2T_6 &= -43 \dots (6'') \end{aligned}$$

Using eqn (2'') as the pivot and subtracting it from eqn eqn (3'')

$$\begin{aligned} (-2/3) (-3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 &= 12) \\ = 2T_2 - T_2 - 2T_4 - T_5 + 2T_6 &= -19 \\ -((2/3) (-3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 &= 12)) \\ = 0T_1 + 2.833333T_2 + (-2T_3) - 4.333333T_4 + 1.333333T_5 &= -11 \end{aligned}$$

for $(4'' - (-\frac{3}{3})(2''))$ (4)

$$= -3T_2 + 11T_3 - 6T_4 - 13T_5 + 6T_6 = -23$$

$$-((- \frac{3}{3})(-3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 = 12))$$

$$= 0T_1 + 4T_2 - 6T_3 - 8T_4 + 7T_5 = -35$$

for $(5'' - (-\frac{2}{3})(2''))$

$$= 2T_2 - 4T_3 + 6T_4 + 10T_5 + 0T_6 = 28$$

$$-((- \frac{2}{3})(-3T_2 - 5T_3 + 0T_4 + (-5)T_5 + 7T_6 = 12)) \checkmark$$

$$= 0T_1 + (-0.66667T_2 + 6T_3 + 6.66667T_4 - 0.66667T_5 = 36$$

for eqn. $(6'' - (-\frac{1}{3})(2''))$

$$-T_2 + 9T_3 - 10T_4 - 15T_5 + 2T_6 = -43$$

$$-((- \frac{1}{3})(-3T_2 + 5T_3 + 0T_4 - 5T_5 - 6T_6 = 12))$$

$$= 0T_1 + 7.333333T_2 + (-10T_3) - 13.33333T_4 + 2.333333T_5 = -47 \checkmark$$

Rewriting the matrix

$$(1''') \quad -T_1 + T_2 - 2T_3 + T_4 + 3T_5 - 6T_6 = 4$$

$$(2''') \quad -0T_1 - 3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 = 12$$

$$(3''') \quad 0T_1 + 0T_2 + 2.333333T_3 - 2T_4 - 4.333333T_5 + 1.333333T_6 = -11$$

$$(4''') \quad -0T_1 + 0T_2 + 4T_3 - 6T_4 - 8T_5 + 7T_6 = -35$$

$$(5''') \quad -0T_1 + 0T_2 - 0.66667T_3 + 6T_4 + 6.66667T_5 - 0.66667T_6 = 36$$

$$(6''') \quad -0T_1 + 0T_2 + 7.333333T_3 - 10T_4 - 13.33333T_5 + 2.333333T_6 = -47$$

Using Eqn (3''') as the pivot

for eqn $(4'''' - (\frac{4}{2.333333})(3'''))$

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Continuation

$$4T_3 - 6T_4 - 8T_5 + 7T_6 = -3$$

$$-\left(\frac{4}{2.333333}\right) \left(2.333333T_3 - 2T_4 - 4.333333T_5 + 1.333333T_6 = -11\right)$$

$$= 0T_3 - 2.57143T_4 - 0.57143T_5 + 4.714286T_6 = 16.14286$$

$$\text{for eqn } \left(5^{III} - \left(\frac{-0.66667}{2.333333}\right) \left(3^{III}\right)\right)$$

$$= -0.66667T_3 + 6T_4 + 6.66667T_5 - 0.66667T_6 = 36$$

$$-\left(\frac{-0.66667}{2.333333}\right) \left(2.333333T_3 - 2T_4 - 4.333333T_5 + 1.333333T_6 = 11\right)$$

$$= 0T_3 + 5.428571T_4 + 5.428571T_5 + 0.28571T_6 = 32.85714$$

$$\text{for eqn } \left(6^{III} - \left(\frac{7.333333}{2.333333}\right) \left(3^{III}\right)\right)$$

$$= 7.333333T_3 - 10T_4 - 13.333333T_5 + 2.333333T_6 = -47$$

$$-\left(-\left(\frac{7.333333}{2.333333}\right) \left(2.333333T_3 - 2T_4 - 4.333333T_5 + 1.333333T_6 = 11\right)\right)$$

$$= 0T_3 - 3.71429T_4 + 0.285714T_5 - 1.85714T_6 = 12.4286$$

Reverting the Matrix

$$(1^{st}) \dots T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$(2^{nd}) \dots 0T_1 - 3T_2 + 5T_3 + 4T_4 - 5T_5 - T_6 = 12$$

$$(3^{rd}) \dots 0T_1 + 0T_2 + 2.333333T_3 - 2T_4 - 4.333333T_5 + 1.333333T_6 = -11$$

$$(4^{th}) \dots 0T_1 + 0T_2 + 0T_3 + -2.57143T_4 - 0.57143T_5 + 4.714286T_6 = 16.14286$$

$$(5^{th}) \dots 0T_1 + 0T_2 + 0T_3 + 5.428571T_4 + 5.428571T_5 - 0.28571T_6 = 32.85714$$

$$(6^{th}) \dots 0T_1 + 0T_2 + 0T_3 - 3.71429T_4 + 0.285714T_5 - 1.85714T_6 = 12.4286$$

⑥

Using eqn 4th as the pivot

$$\text{for eqn } (5^{th}) - \left(\frac{5.428571}{-2.57143} \right) (4^{th})$$

$$= 5.428571T_4 + 5.428571T_5 - 0.28571T_6 = 32.85714$$

$$- \left(\frac{-5.428571}{2.57143} \right) (2.57143T_4 - 0.57143T_5 + 4.714286T_6 = -16.1429)$$

$$= 0T_4 + 4.222222 + 9.666667 = -1.22222$$

$$\text{for eqn } (6^{th}) - \left(\frac{-3.71429}{-2.57143} \right) (4^{th})$$

$$= -3.71429T_4 + 0.285714T_5 - 1.85714T_6 = -12.4286$$

$$- \left(\frac{-3.71429}{-2.57143} \right) (-2.57143T_4 - 0.57143T_5 + 4.714286T_6 = -16.1429)$$

$$= 0T_4 + 1.111111T_5 + (-8.66667T_6) = 10.88889$$

~~Re eqn~~ Rewriting the Above matrix

$$\begin{aligned} (1^{st}) \dots & T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ (2^{nd}) \dots & 0T_1 - 3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 = 12 \\ (3^{rd}) \dots & 0T_1 + 0T_2 - 2.333333T_3 - 2T_4 - 4.333333T_5 + 1.333333T_6 = -11 \\ (4^{th}) \dots & 0T_1 + 0T_2 + 0T_3 - 2.57143T_4 - 0.57143T_5 + 4.714286T_6 = -16.1429 \\ (5^{th}) \dots & 0T_1 + 0T_2 + 0T_3 + 0T_4 + 4.222222T_5 + 9.666667T_6 = -1.22222 \\ (6^{th}) \dots & 0T_1 + 0T_2 + 0T_3 + 0T_4 + 1.111111T_5 - 8.66667T_6 = 10.88889 \end{aligned}$$

Using equation 5th as pivot

$$\text{for eqn } (6^{th}) - \left(\frac{1.111111}{4.222222} \right) (5^{th})$$

$$= (1.111111T_5 - 8.66667T_6) - \left(\frac{1.111111}{4.222222} \right) (4.222222T_5 + 9.666667T_6 = -1.22222)$$

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$$= 0T_5 - 11.2105T_6 = 11.21053 \quad \dots \star$$

From eqn \star

$$11 = -11.2105T_6 = 11.21053$$

$$T_6 = \frac{11.21053}{-11.2105}$$

$$T_6 = -1$$

Substituting T_6 into eqn (5th)

$$4.222222T_5 + 9.666667(T_6) = -1.22222$$

$$4.222222T_5 + 9.666667(-1) = -1.22222$$

$$4.222222T_5 - 9.666667 = -1.22222$$

$$T_5 = \frac{-1.22222 + 9.666667}{4.222222}$$

$$T_5 = 2$$

and T_5
Substituting T_6 and T_5 into eqn (4th)

$$-2.57143T_4 + (-0.57143T_5 + 4.71426T_6) = -16.1429$$

$$-2.57143T_4 - 0.57143(2) + 4.71426(-1) = -16.1429$$

$$-2.57143T_4 - 1.14286 + 4.71426 = -16.1429$$

$$T_4 = \frac{-16.1429 + 1.14286 + 4.71426}{-2.57143} = 4$$

②

Substituting T_4, T_5 and T_6 into Eqn (3ⁱⁱⁱ)

$$2.33333T_3 - 2T_4 - 4.33333T_5 + 1.33333T_6 = -11$$

$$-2.33333T_3 - 2(4) - 4.33333(2) + 1.33333(-1) = -11$$

$$-2.33333T_3 - 8 - 8.66666 - 1.33333 = -11$$

$$T_3 = \frac{-11 + 8 + 8.66666}{-2.33333}$$

$$= 3$$

Substituting T_3, T_4, T_5, T_6 into Eqn 2⁽ⁱⁱ⁾

$$-3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 = 12$$

$$= -3T_2 + 5(3) + 0(4) - 5(2) - (-1) = 12$$

$$-3T_2 + 15 + 0 - 10 + 1 = 12$$

$$T_2 = \frac{12 - 6}{-3}$$

$$T_2 = -2$$

Substituting T_2, T_3, T_4, T_5 and T_6 into eqn (1)

$$T_1 + T_2 + (-2)T_3 + T_4 + 3T_5 - T_6 = 4$$

$$= T_1 - 2 + (-2)(3) + 4 + 3(2) - (-1) = 4$$

$$= T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 = 4 - 3$$

$$= 1$$

Hence the Values of T_1, T_2, T_3, T_4, T_5 and T_6 are 1, -2, 3, 4, 2, -1 respectively