$$\bar{I}_1 + \bar{I}_2 - 2\bar{I}_3 + \bar{I}_4 + 3\bar{I}_5 - \bar{I}_6 = 4$$

$$2\bar{I}_1 - \bar{I}_2 + \bar{I}_3 + 2\bar{I}_4 + \bar{I}_5 - 3\bar{I}_6 = 20$$

$$\bar{I}_1 + 3\bar{I}_2 - 3\bar{I}_3 - \bar{I}_4 + 2\bar{I}_5 + \bar{I}_6 = -15$$

$$5\bar{I}_1 + 2\bar{I}_2 - \bar{I}_3 - \bar{I}_4 + 2\bar{I}_5 + \bar{I}_6 = -3$$

$$-3\bar{I}_1 - \bar{I}_2 + 2\bar{I}_3 + 3\bar{I}_4 + \bar{I}_5 + 3\bar{I}_6 = 16$$

$$4\bar{I}_1 + 3\bar{I}_2 + \bar{I}_3 - 6\bar{I}_4 - 3\bar{I}_5 - 2\bar{I}_6 = -27$$

solution

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & 6 & -3 & -2 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \\ \bar{I}_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

Using Gaus elimination method

$F_1 = 2 \quad F_2 = 1 \quad F_3 = 5 \quad F_4 = -3 \quad F_5 = 4$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2-2(1) & -1-2(1) & 1-2(-2) & 2-2(1) & 1-2(3) & -3-2(-1) \\ 1-1(1) & 3-1(1) & -3-1(-2) & -1-1(1) & 2-1(3) & 1-1(-1) \\ 5-5(1) & 2-5(1) & -1-5(-2) & -1-5(1) & 2-5(3) & 1-5(-1) \\ -3+3(1) & -1+3(1) & 2+3(-2) & 3+3(1) & 1+3(3) & 3+3(-1) \\ 4-4(1) & 3-4(1) & 1-4(-2) & 6-4(1) & -3-4(3) & -2-4(-1) \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \\ \bar{I}_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20-2(4) \\ -15-1(4) \\ -3-5(4) \\ 16+3(4) \\ -27-4(4) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 9 & 10 & -15 & 2 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \\ \bar{I}_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{bmatrix}$$

$F_1^* = -2/3$ $F_2^* = -1$ $F_3 = -2/3$ $F_4 = 1/3$

$$\begin{bmatrix} 1 & 2 & -2 & 1 & 3 & 0 & -1 \\ 0 & -3 & 5 & 0 & -5 & 0 & -1 \\ 0 & 2+2/3(-3) & -1+2/3(5) & -2+2/3(0) & -1+2/3(-5) & 0 & 2+2/3(-1) \\ 0 & -3-1(-3) & 9-1(5) & -6-1(0) & -13-1(-5) & 0 & 6-1(-1) \\ 0 & 2+2/3(-3) & 4+2/3(5) & 6+2/3(0) & 10+2/3(-5) & 0 & 0+2/3(-1) \\ 0 & -1-1/3(-3) & 9-1/3(5) & -10-1/3(0) & -15-1/3(-5) & 0 & 2-1/3(-1) \end{bmatrix} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \\ \bar{T}_4 \\ \bar{T}_5 \\ \bar{T}_6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \\ -19+2/3(12) \\ -23-1(12) \\ 28+2/3(12) \\ -43-1/3(12) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -2/3 & 6 & 20/3 & -2/3 \\ 0 & 0 & 22/3 & -10 & -40/3 & 7/3 \end{bmatrix} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \\ \bar{T}_4 \\ \bar{T}_5 \\ \bar{T}_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ 47 \end{bmatrix}$$

$F_1 = 12/7$ $F_2 = -2/7$ $F_3 = 22/7$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 4-12/7(7/3) & -6-12/7(-2) & -8-12/7(-13/3) & 7-12/7(4/3) \\ 0 & 0 & -2/3+2/7(7/3) & 6+2/7(-2) & 20/3+2/7(-13/3) & -2/3+2/7(4/3) \\ 0 & 0 & 22/3-22/7(7/3) & -10-22/7(-2) & -40/3-22/7(-13/3) & 7/3-22/7(4/3) \end{bmatrix} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \\ \bar{T}_4 \\ \bar{T}_5 \\ \bar{T}_6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35-12/7(-4) \\ 36+2/7(-11) \\ -47-22/7(-11) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 38/7 & 38/7 & -2/7 \\ 0 & 0 & 0 & -26/7 & 2/7 & -13/7 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ 280/7 \\ -87/7 \end{bmatrix}$$

$F_1 = \frac{-19}{9}$ $\quad F_2 = \frac{13}{9}$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/9 & 33/7 \\ 0 & 0 & 0 & 38/7 + 19/9(-18/7) & 38/9 + 19/9(-4/7) & -2/7 + 19/9(33/7) \\ 0 & 0 & 0 & -26/7 - 13/9(-18/7) & 2/7 - 13/9(-4/7) & -13/9 - 13/9(33/7) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ 280/7 + 19/9(-113/7) \\ -87/7 - 13/9(-113/7) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 10/9 & -26/3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 98/9 \end{bmatrix}$$

$F_1 = 5/19$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 10/9 - 5/19\left(\frac{38}{9}\right) & -\frac{26}{3} - \frac{5}{19}\left(\frac{29}{3}\right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 98/9 - 5/19(-11/9) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 0 & -213/19 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 213/19 \end{bmatrix}$$

$$-\frac{213}{19} T_6 = \frac{213}{19}$$

$$T_6 = 213/19 \div -\frac{213}{19}$$

$$T_6 = -1$$

$$\frac{38}{9} T_5 + \frac{29}{3} T_6 = -\frac{11}{9} \qquad T_5 = \left(-\frac{11}{9} - \frac{29}{3} \times -1\right) \div \frac{38}{9}$$

$$T_5 = 2$$

$$T_4 = \left(-\frac{113}{7} + \frac{8}{7} + \frac{33}{7}\right) \times -\frac{7}{18} = 4$$