Question 1 [20 Marks]

If the model of a system having thermocouples measuring temperatures, T (oC), at its different points is given by the set of expressions in Equation (1), estimate the values of the temperatures using:

……………..Equation (1)

 T1+T2-2T3+T4+3T5-T6=4

2T1-T2+T3+2T4+T5-3T6=20

 T1+3T2-3T3-T4+2T5+T6= -15 ……………..Equation (1)

5T1+2T2-T3-T4+2T5+T6= -3

-3T1-T2+2T3+3T4+T5+3T6= 16

 4T1+3T2+T3-6T4-3T5-2T6= -27

(a) Gauss elimination method manually (with the aid of calculator),

(b) Gauss elimination method with the aid of Microsoft Excel,

(c) Gauss elimination method with the aid of MATLAB,

(d) matrix inverse method with the aid of Microsoft Excel, and

(e) matrix inverse method with the aid of MATLAB.

**Solution**

**A. Solution Gauss elimination method manually (with the aid of calculator)**

 T1+T2-2T3+T4+3T5-T6=4

2T1-T2+T3+2T4+T5-3T6=20

 T1+3T2-3T3-T4+2T5+T6= -15

5T1+2T2-T3-T4+2T5+T6= -3 ……………..Equation (1)

-3T1-T2+2T3+3T4+T5+3T6= 16

 4T1+3T2+T3-6T4-3T5-2T6= -27

The matrix forms of Equation 1 are as follows:

 1 1 -2 1 3 -1 T1 4

 2 - 1 1 2 1 -3 T2 20

 1 3 -3 -1 2 1 T3 = -15

 5 2 -1 -1 2 1 T4 -3

 -3 -1 2 3 1 3 T5 16

 4 3 1 -6 -3 -2 T6 -27

By subtracting the product of the first pivot element of each column of row one and the pivot factor of each row from each element of rows 2 to 6, we get the following:

 Pivot factor

 1 1 -2 1 3 -1 T1 4 f12=(-2/1)=2

 0 -3 5 0 -5 -1 T2 12 f13=(1/1)=1

 0 2 -1 -2 -1 2 T3 = -19 f14=(5/1)=5

 0 -3 9 -6 -13 6 T4 -23 f15=(-3/1)=-3

 0 2 -4 6 10 0 T5 28 f16=(4/1)=4

 0 1 9 -10 -15 2 T6 -43

By subtracting the product of the second pivot element of each column of row two and the pivot factor of each row from each element of rows 3 to 6, we get the following:

 1 1 -2 1 3 -1 T1 4

 0 -3 5 0 -5 -1 T2 12 f23=(2/-3)=1

 0 0 7/3 -2 -13/3 4/3 T3 = -11 f24=(-3/-3)=1

 0 0 4 -6 -8 7 T4 -35 f25=(2/-3)=-2/3

 0 0 -2/3 6 20/3 -2/3 T5 36 f26=(4/1)=4

 0 0 -22/3 -10 -40/3 7/3 T6 -47

By subtracting the product of the third pivot element of each column of row three and the pivot factor of each row from each element of rows 4 to 6, we get the following:

 1 1 -2 1 3 -1 T1 4

 0 -3 5 0 -5 -1 T2 12

 0 0 7/3 -2 -13/3 4/3 T3 = -11 f34=(-4/(7/3)=12/7

 0 0 0 -18/7 -4/7 33/7 T4 -113/7 f35=(2/-3)/(7/3)= -2/7

 0 0 0 38/7 38/7 -2/7 T5 230/7 f36=(22/3)/(7/3)=22/7

 0 0 0 -26/7 2/7 -13/7 T6 -87/7

By subtracting the product of the fourth pivot element of each column of row four and the pivot factor of each row from each element of rows 5 to 6, we get the following:

 1 1 -2 1 3 -1 T1 4

 0 -3 5 0 -5 -1 T2 12

 0 0 7/3 -2 -13/3 4/3 T3 = -11

 0 0 0 -18/7 -4/7 33/7 T4 -113/7 f45=(38/7)/(-18/7)= -19/9

 0 0 0 0 38/9 29/3 T5 -11/9 f46=(-26/7)/(-18/7)=13/9

 0 0 0 0 10/9 -26/3 T6 -98/9

By subtracting the product of the fourth pivot element of each column of row fifth and the pivot factor of each row from each element of rows 6, we get the following:

 1 1 -2 1 3 -1 T1 4

 0 -3 5 0 -5 -1 T2 12

 0 0 7/3 -2 -13/3 4/3 T3 = -11

 0 0 0 -18/7 -4/7 33/7 T4 -113/7

 0 0 0 0 38/9 29/3 T5 -11/9 f56=(10/9)/(38/9)=5/9

 0 0 0 0 10/9 -26/3 T6 213/19

Hence,(-213/19)T6=213/19

T6 = -1

(38/9)T5 +(29/3)T6= -11/9

T5 = ((-11/9)-(29/3)(-1)/(38/9)) = 2

-18/7T4 + -4/7T5+33/7T6=-113/7

T4 = (( -113/7-(4/7(2)-33/7(-1)))/(-18/7))=4

( 7/3)T3 + (-2) T4 +(-13/3) T5 + (4/3) T6 = -11

T3 = ((-11 – ((-2)( (4) +(-13/3) ( (2) + (4/3) (-1)))/(7/3 ) = 3

-3T2 + 5T3 + (0)T4 - 5T5 - T6 = 12

T2 = ((12 – (5(3) + (0)(4) - 5(2) – (-1)))/-3) = -2

T1 + T2 -2T3 + T4 + 3T5 - T6 = 4

T1 = ((4 – ((-2) -2(3) + (4) + 3(2) – (-1)))/1) = 1

**Therefore, the solution of the sets of equations in matrix form is:**

 T1 1

 T2 -2

 T3 = 3

 T4 4

 T5 2

 T6 -1

B. **Solution using Gauss elimination method with the aid of Microsoft Excel**







 







