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Petroleum Engineering

## Assignment II

If the maximum percentage absolute error is desired to be  $10^{-9}$ . Using the Newton-Raphson iteration method and initial guess value of  $x_0 = 0.5$ . Find the root of the function in the given equation

i) Manually

ii) With the aid of MATLAB

$$f(x) = e^{-0.5x}(4-x) - 2$$

NB For the manual solution, use all the values given by the calculator

### Solution

$$f(x) = e^{-0.5x}(4-x) - 2$$

$$f'(x) =$$

$$\text{Let } u = e^{-0.5x}, \quad v = (4-x)$$

$$du = -0.5e^{-0.5x} dx, \quad dv = -1 dx$$

$$f'(x) = u dv + v du$$

$$= -e^{-0.5x} - 0.5e^{-0.5x}(4-x)$$

$$x_0 = 0.5 \text{ (initial guess)}$$

General Newton-Raphson's Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x_0) = f(0.5) = 0.7258027167$$

$$f'(x_0) = f'(0.5) = -2.14762153$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.8388906064 \text{ (Root 1)}$$

$$f(x_1) = 0.07814929779$$

$$f'(x_1) = -1.696486032$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.8849560003 \text{ (Root 2)}$$

$$f(x_2) = 1.236575203 \times 10^{-3}$$

$$f'(x_2) = 1.643060762$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.885708605 \text{ (root 3)}$$

$$f(x_3) = 3.23583557 \times 10^{-9}$$

$$f'(x_3) = -1.642200929$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.885708802 \text{ (root 4)}$$

$$f(x_4) = 7.845 \times 10^{-12}$$

$$f'(x_4) = -1.642200704$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.885708802 \text{ (root 5)}$$

$\therefore 0.885708802$  is the root of eqn (1)

### MATLAB

[ ] function [x1, 200, reterr] = assign2(x0, maxc1, tol, iter, f, fprime)

- $x_0 = 0.5$
- $\text{maxc1} = 100$
- $\text{tol} = 0.000000001$
- $\text{iter} = 1$
- $f = @(x) [\exp(-0.5 * x) * (4 - x)] - 2;$
- $f_{\text{prime}} = @(x) [\exp(-0.5 * x) + (0.05 * \exp(0.05 * x) * (4 - x))]$
- For  $i = 1$  to  $\text{maxc1}$ 
  - $x_1 = x_0 - f(x_0) / f_{\text{prime}}(x_0);$
  - $\text{err} = \text{abs}(x_1 - x_0); \text{reterr} = \text{abs}(x_1 - x_0) / x_1;$
  - $\text{fprintf}(\%2 (\%10.10f \%10.10f \%10.10f \%10.10f) x_0, x_1, \text{err}, \text{reterr})$
  - $x_0 = x_1; \text{iter} = \text{iter} + 1;$
  - $\text{if err} < \text{tol}; \text{break};$
  - $\text{break}; \text{end}$