Elect/Elect NWANKWO NWACHUKWU 16/ENG04/036

The model of a system having thermocouples measuring Temperatures, T(°C), at its different point is given by the equ 1, estimate the values of the Temperatures using:

a) Gauss elimination method manually (calc)
b) Gauss elimination method Excel
c) " " " MathLab
d) matrix inverse method Excel
e) " " " MathLab.

$$\begin{cases} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27 \end{cases}$$

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 + a_{36}x_6 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 + a_{46}x_6 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 + a_{56}x_6 \\ a_{61}x_1 + a_{62}x_2 + a_{63}x_3 + a_{64}x_4 + a_{65}x_5 + a_{66}x_6 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

$$Ax = T$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ 15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
$a_{21} - \frac{a_{21}}{a_{11}} \cdot a_{11}$	$a_{22} - \frac{a_{21}}{a_{11}} \cdot a_{12}$	$a_{23} - \frac{a_{21}}{a_{11}} \cdot a_{13}$	$a_{24} - \frac{a_{21}}{a_{11}} \cdot a_{14}$	$a_{25} - \frac{a_{21}}{a_{11}} \cdot a_{15}$	$a_{26} - \frac{a_{21}}{a_{11}} \cdot a_{16}$
$a_{31} - \frac{a_{31}}{a_{11}} \cdot a_{11}$	$a_{32} - \frac{a_{31}}{a_{11}} \cdot a_{12}$	$a_{33} - \frac{a_{31}}{a_{11}} \cdot a_{13}$	$a_{34} - \frac{a_{31}}{a_{11}} \cdot a_{14}$	$a_{35} - \frac{a_{31}}{a_{11}} \cdot a_{15}$	$a_{36} - \frac{a_{31}}{a_{11}} \cdot a_{16}$
$a_{41} - \frac{a_{41}}{a_{11}} \cdot a_{11}$	$a_{42} - \frac{a_{41}}{a_{11}} \cdot a_{12}$	$a_{43} - \frac{a_{41}}{a_{11}} \cdot a_{13}$	$a_{44} - \frac{a_{41}}{a_{11}} \cdot a_{14}$	$a_{45} - \frac{a_{41}}{a_{11}} \cdot a_{15}$	$a_{46} - \frac{a_{41}}{a_{11}} \cdot a_{16}$
$a_{51} - \frac{a_{51}}{a_{11}} \cdot a_{11}$	$a_{52} - \frac{a_{51}}{a_{11}} \cdot a_{12}$	$a_{53} - \frac{a_{51}}{a_{11}} \cdot a_{13}$	$a_{54} - \frac{a_{51}}{a_{11}} \cdot a_{14}$	$a_{55} - \frac{a_{51}}{a_{11}} \cdot a_{15}$	$a_{56} - \frac{a_{51}}{a_{11}} \cdot a_{16}$
$a_{61} - \frac{a_{61}}{a_{11}} \cdot a_{11}$	$a_{62} - \frac{a_{61}}{a_{11}} \cdot a_{12}$	$a_{63} - \frac{a_{61}}{a_{11}} \cdot a_{13}$	$a_{64} - \frac{a_{61}}{a_{11}} \cdot a_{14}$	$a_{65} - \frac{a_{61}}{a_{11}} \cdot a_{15}$	$a_{66} - \frac{a_{61}}{a_{11}} \cdot a_{16}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ 1 \\ x_5 \\ 1 \\ x_6 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 - \frac{a_{21}}{a_{11}} b_1 \\ b_3 - \frac{a_{31}}{a_{11}} b_1 \\ b_4 - \frac{a_{41}}{a_{11}} b_1 \\ \\ b_5 - \frac{a_{51}}{a_{11}} b_1 \\ \\ b_6 - \frac{a_{61}}{a_{11}} b_1 \end{bmatrix}$$

1st pivot

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 0 & -3 & +4 & -6 & 13 & 6 \\ 0 & 2 & -4 & 5 & 10 & 0 \\ 0 & -19 & -10 & -15 & 2 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{bmatrix}$$

$f_{12} = 2$

$f_{13} = 1$

$f_{14} = 5$

$f_{15} = -3$

$f_{16} = 4$

2nd Pivot

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & \frac{18}{3} & \frac{4}{3} \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -\frac{2}{3} & 6 & \frac{20}{3} & -\frac{2}{3} \\ 0 & 0 & \frac{22}{3} & -10 & -\frac{40}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ -47 \end{bmatrix}$$

$$\begin{vmatrix} F_{23} = 2 \\ F_{24} = 1 \\ F_{25} = -\frac{2}{3} \\ F_{26} = \frac{1}{3} \end{vmatrix}$$

$\therefore$ ~~3rd~~ Pivot

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{33}{7} \\ 0 & 0 & 0 & \frac{38}{7} & \frac{38}{7} & -\frac{2}{7} \\ 0 & 0 & 0 & -\frac{26}{7} & \frac{2}{7} & -\frac{13}{7} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ \frac{230}{7} \\ -\frac{87}{7} \end{bmatrix}$$

$$\begin{bmatrix} F_{34} = \left(\frac{4}{7/3}\right) = \frac{12}{7} \\ F_{35} = \frac{-2}{7} \\ F_{36} = \frac{22}{7} \end{bmatrix}$$

4th Pivot

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{33}{7} \\ 0 & 0 & 0 & 0 & \frac{38}{9} & \frac{33}{7} \\ 0 & 0 & 0 & 0 & \frac{38}{9} & \frac{29}{3} \\ 0 & 0 & 0 & 0 & \frac{10}{9} & \frac{26}{3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -1 \\ -\frac{113}{7} \\ -\frac{11}{9} \\ \frac{98}{9} \end{bmatrix}$$

$$F_{45} = -\frac{19}{9}$$

$$F_{46} = \frac{13}{9}$$

Fifth Pivot

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & 4/3 \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 0 & -\frac{213}{19} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ -\frac{11}{9} \\ \frac{213}{19} \end{bmatrix}$$

$T_6 = \frac{5}{19}$

Hence, $\frac{-213}{19} T_6 = \frac{213}{19}$

$T_6 = -1$

$\frac{38}{9} T_5 + \frac{29}{3} T_6 = \frac{-11}{9}$

$\therefore T_5 = \left[\left(\frac{-11}{9} - \frac{29}{3}(-1)\right) / \frac{38}{9}\right] = 2$

$T_5 = 2$

$T_4 = \left(\left(-\frac{113}{7} - \left(\frac{4}{7}(2) - \frac{33}{7}(-1)\right)\right) / \left(-\frac{18}{7}\right)\right)$

$\therefore T_4 = 4$

$$\left(\frac{7}{3}\right)T_3 + (-2)T_4 + \left(-\frac{13}{3}\right)T_5 + \left(\frac{4}{3}\right)T_6 = -11$$

$$\therefore T_3 = \left(\left(-11 - (-2)(4) + \left(-\frac{13}{3} \times 2\right) + \frac{4}{3} \times -1\right)\right) / \frac{7}{3}$$

$$\therefore \underline{T_3 = 3}$$

$$\therefore \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 2 \\ -1 \end{bmatrix}$$