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Course: ENG 382 - Eng. Maths DV

Solve using Naive Gauss Elimination:

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$

$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$

$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27$$

Soln

representing in matrix form,

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

$$R_1 = 2, \quad F_2 = 1, \quad F_3 = 5, \quad R_4 = -3, \quad F_5 = 4$$

Subst. into the matrix,

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2-2(1) & -1-2(1) & 1-2(-2) & 2-2(1) & 1-2(3) & -3-2(-1) \\ 1-1(1) & 3-1(1) & -3-1(-2) & -1-1(1) & 2-1(3) & 1-1(-1) \\ 5-5(1) & 2-5(1) & -1-5(-2) & -1-5(1) & 2-5(3) & 1-5(1) \\ -3+3(1) & -1+3(1) & 2+3(-2) & 3+3(1) & 1+3(3) & -3+3(-1) \\ 4-4(1) & 3-4(1) & 1-4(-2) & -6-4(1) & -3-4(3) & -2-4(-1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20-2(4) \\ -15+1(4) \\ -3-5(4) \\ 16+3(4) \\ -27+1(4) \end{bmatrix}$$

Solving the above matrix yields:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 9 & 10 & -15 & 2 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \\ \Gamma_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{bmatrix}$$

solving this gives:

$$F_{12} = -2/3, F_{22} = -1, P_3 = -2/3, F_{42} = 1/3$$

subst. into the matrix,

$$\begin{bmatrix} 1 & 2 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 + 2/3(-3) & -1 + 2/3(5) & -2 + 2/3(0) & -1 + 2/3(-5) & 2 + 2/3(-1) \\ 0 & -3 - 1(-3) & 9 - 1(5) & -6 - 1(0) & -13 - 1(-5) & 6 - 1(-1) \\ 0 & 2 + 2/3(-3) & 4 + 2/3(5) & 6 + 2/3(0) & 10 + 2/3(-5) & 0 + 2/3(-1) \\ 0 & -1 - 1/3(-3) & 9 - 1/3(5) & -10 + 1/3(0) & -15 - 1/3(-5) & 2 - 1/3(-1) \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \\ \Gamma_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 + 2/3(12) \\ -23 - 1(12) \\ 28 + 2/3(12) \\ -43 - 1/3(12) \end{bmatrix}$$

solving the matrix yields

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -2/3 & 6 & 20/3 & -2/3 \\ 0 & 6 & 22/3 & -10 & -40/3 & 7/3 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \\ \Gamma_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ 47 \end{bmatrix}$$

solving the matrix gives

$$P_1 = 12/7, P_2 = -2/7, P_3 = 22/7$$

subst. into the matrix above

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 4 - 12/7(7/3) & -6 - 12/7(-2) & -8 - 12/7(-13/3) & 7 - 12/7(4/3) \\ 0 & 0 & -2/3 + 2/7(7/3) & 6 + 2/7(-2) & 20/3 + 2/7(-13/3) & -2/3 + 2/7(4/3) \\ 0 & 0 & 22/3 - 22/7(7/3) & -10 - 22/7(-2) & -40/3 + 22/7(-13/3) & 7/3 - 22/7(4/3) \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \\ \Gamma_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 + 12/7(-11) \\ 36 + 2/7(-11) \\ 47 - 22/7(-11) \end{bmatrix}$$

solving the matrix yields

4	1	1	-2	1	3	-1	T_1	4
12	0	-3	5	0	-5	-1	T_2	12
-19	0	0	$\frac{7}{3}$	-2	$-\frac{13}{3}$	$\frac{4}{3}$	T_3	-11
-23	0	0	0	$-\frac{18}{7}$	$-\frac{4}{7}$	$\frac{33}{7}$	T_4	$-\frac{43}{7}$
28	0	0	0	$\frac{38}{7}$	$\frac{38}{7}$	$-\frac{2}{7}$	T_5	$\frac{280}{7}$
-43	0	0	0	$-\frac{26}{7}$	$\frac{2}{7}$	$-\frac{13}{7}$	T_6	$-\frac{87}{7}$

Solving the matrix gives

$$P_1 = -\frac{19}{9}, \quad P_2 = \frac{13}{9}$$

T_1	4	1	1	-2	1	3	-1	T_1	4
T_2	12	0	-3	5	0	-5	-1	T_2	12
T_3	$-19 + \frac{7}{3}(12)$	0	0	$\frac{7}{3}$	-2	$-\frac{13}{3}$	$\frac{4}{3}$	T_3	-11
T_4	$-23 - 1(12)$	0	0	0	$-\frac{18}{7}$	$-\frac{4}{7}$	$\frac{33}{7}$	T_4	$-\frac{43}{7}$
T_5	$28 + \frac{2}{3}(12)$	0	0	0	$\frac{38}{7} + \frac{1}{7}(-\frac{18}{7})$	$\frac{38}{7} + \frac{1}{7}(-\frac{4}{7})$	$-\frac{2}{7} + \frac{1}{7}(\frac{33}{7})$	T_5	$\frac{280}{7} + \frac{1}{7}(-\frac{43}{7})$
T_6	$-43 - \frac{1}{3}(12)$	0	0	0	$-\frac{26}{7} - \frac{1}{7}(-\frac{18}{7})$	$\frac{2}{7} - \frac{1}{7}(-\frac{4}{7})$	$-\frac{13}{7} - \frac{1}{7}(\frac{33}{7})$	T_6	$-\frac{87}{7} - \frac{1}{7}(\frac{43}{7})$

Solving the matrix yields

4	1	1	-2	1	3	-1	T_1	4
12	0	-3	5	0	-5	-1	T_2	-12
-11	0	0	$\frac{7}{3}$	-2	$-\frac{13}{3}$	$\frac{4}{3}$	T_3	-11
-35	0	0	0	$-\frac{18}{7}$	$-\frac{4}{7}$	$\frac{33}{7}$	T_4	$-\frac{43}{7}$
36	0	0	0	0	$\frac{38}{9}$	$\frac{29}{3}$	T_5	$-\frac{4}{9}$
7	0	0	0	0	$\frac{10}{9}$	$-\frac{26}{3}$	T_6	$\frac{98}{9}$

Solving the matrix gives

$$P_1 = \frac{5}{19}$$

subst. into the matrix

T_1	4	1	1	-2	1	3	-1	T_1	4
T_2	12	0	-3	5	0	-5	-1	T_2	12
T_3	-11	0	0	$\frac{7}{3}$	-2	$-\frac{13}{3}$	$\frac{4}{3}$	T_3	-11
T_4	$-35 + \frac{2}{3}(-11)$	0	0	0	$-\frac{18}{7}$	$-\frac{4}{7}$	$\frac{33}{7}$	T_4	$-\frac{43}{7}$
T_5	$36 + \frac{2}{3}(-11)$	0	0	0	0	$\frac{38}{9}$	$\frac{29}{3}$	T_5	$-\frac{4}{9}$
T_6	$7 - \frac{2}{3}(-11)$	0	0	0	0	$\frac{10}{9} - \frac{5}{9}(\frac{38}{9})$	$-\frac{26}{3} - \frac{5}{9}(\frac{29}{3})$	T_6	$\frac{98}{9} - \frac{5}{9}(-\frac{4}{9})$

Solving the matrix yields:

$$\begin{bmatrix} 1 & 1 & -2 & 11 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 83/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 0 & -213/19 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \\ \Gamma_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 213/19 \end{bmatrix}$$

Solving for $\Gamma_6, \Gamma_5, \Gamma_4, \Gamma_3, \Gamma_2$ and Γ_1

$$-\frac{213}{19} \Gamma_6 = \frac{213}{19}$$

$$\therefore \Gamma_6 = \frac{213}{19} \times -\frac{19}{213} = -1 //$$

$$\frac{38}{9} \Gamma_5 + \frac{29}{3} \Gamma_6 = -\frac{11}{9}$$

$$\therefore \Gamma_5 = \left(-\frac{11}{9} + \frac{29}{3} \right) \times \frac{9}{38} = 2 //$$

$$-\frac{18}{7} \Gamma_4 - \frac{4}{7} \Gamma_5 + \frac{33}{7} \Gamma_6 = -11$$

$$\therefore \Gamma_4 = \left(\frac{-113}{7} + \frac{8}{7} + \frac{23}{7} \right) \times -\frac{7}{18} = 4 //$$

$$\Gamma_3 = (-11 + 8 + \frac{26}{3} + \frac{4}{3}) \times \frac{3}{7} = 3 //$$

$$\Gamma_2 = (12 - 15 + 10 - 1) \times \frac{1}{3} = -2 //$$

$$\Gamma_1 = (4 + 2 + 6 - 4 - 6 - 1) = 1 //$$

$$\therefore \begin{aligned} \Gamma_1 &= 1 \\ \Gamma_2 &= -2 \\ \Gamma_3 &= 3 \\ \Gamma_4 &= 4 \\ \Gamma_5 &= 2 \\ \Gamma_6 &= -1 \end{aligned}$$