**QUESTION 1 [20 MARKS]**

If the maximum percentage absolute error is desired to be 1E-9, using the Newton-Raphson iteration method and initial guess value of 0.5, find the root of the function given in Equation (1.1)

(a) manually, and

(b) with the aid of MATLAB.

f(x) = $e^{-0.5x}$x (4-x) -2 ……………………………………Equation (1.1)

NB: For the manual solution, use all the values given by the calculator.

**Solution**

**(a) Manual solution**

f(x) =$e^{-0.5x}$ \*(4-x) -2

f’(x) = $e^{-0.5x}$ (-1)-(4-x) \* 0.5 X\*$e^{-0.5x}$= -$e^{-0.5x}$ \*(1+0.5 \*(4-x))

f’(x) = - $e^{-0.5x}$\* (3-0.5x)

Applying the Newton-Raphson iteration method,

xi+1 = xi – (F(xi)/F’(xi))

using an initial guess of x =0.5,

xi+1 = 0.5 – ($e^{-0.5(0.5)}$\* (4-(0.5)) -2)/ (-$e^{-0.5(0.5)}$ \* (3-0.5(0.5)))) = 0.838890606

xi+1= 0.838890606–($e^{-0.5(0.838890606)}$ \*(4-(0.838890606))-2)/($ e^{-0.5(0.838890606)}$\*(3-0.5(0.838890606)))) = 0.8849560003

xi+1=0.8849560003–( $e^{-0.5(0.8849560003}$\*(4-(0.8849560003))-2)/(-$ e^{-0.5(0.8849560003)}$ \*(3-0.5(0.8849560003)))) = 0.885708605

xi+1=0.885708605–( ($e^{-0.5(0.885708605)}$\*(4-(0.885708605))-2)/(- $e^{-0.5(0.885708605)}$\*(3-0.5(0.885708605)))) = 0.885708802

xi+1=0.885708802–( $e^{-0.5(0.885708802)}$\*(4-(0.885708802))-2)/(-$ e^{-0.5(0.885708802)}$ \*(3-0.5(0.885708802)))) = 0.885708802

The absolute error is given as:

Ea =│(xi+1 - xi ) / xi+1│\* 100

Ea = │(0.838890606 - 0.5 ) / 0.838890606│X 100 = 40.39747299

Ea = │(0.8849560003- 0.838890606) / 0.8849560003│X 100 = 5.205388097

Ea = │(0.885708605- 0.8849560003) / 0.885708605│X 100 = 0.08497204337

Ea = │(0.885708802- 0.885708605) / 0.885708802│X 100 = 2.224207319 \*10-5

Ea = │(0.885708802- 0.885708802) / 0.885708802│X 100 = 0

|  |  |  |
| --- | --- | --- |
| In tabular form the results are: i  | Xi+1  | Ea  |
| 0  | 0.5  | -  |
| 1  | 0.838890606  | 40.39747299  |
| 2  | 0.8849560003  | 5.205388097  |
| 3  | 0.885708605  | 0.08497204337  |
| 4  | 0.885708802  | 2.224207319 \*10-5  |
| 5  | 0.885708802  | 0  |

