

Example 3

$2x^2 + 3x + 2$	$2x^2 + 3x + 2$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$x^2 + x + 1$	$x^2 + x + 1$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$0x^2 - x + 0$	$0x^2 - x + 0$
$+(x^2 + 2x + 1)$	$+(x^2 + 2x + 1)$
$0x^2 + x + 1$	$0x^2 + x + 1$

Long Division (Using Synthetic)

$2x^2 + 3x + 2$	$2x^2 + 3x + 2$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$x^2 + x + 1$	$x^2 + x + 1$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$0x^2 - x + 0$	$0x^2 - x + 0$
$+(x^2 + 2x + 1)$	$+(x^2 + 2x + 1)$
$0x^2 + x + 1$	$0x^2 + x + 1$

Example 4 through 6

$2x^2 + 3x + 2$	$2x^2 + 3x + 2$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$x^2 + x + 1$	$x^2 + x + 1$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$0x^2 - x + 0$	$0x^2 - x + 0$
$+(x^2 + 2x + 1)$	$+(x^2 + 2x + 1)$
$0x^2 + x + 1$	$0x^2 + x + 1$

Example 4 through 6

$2x^2 + 3x + 2$	$2x^2 + 3x + 2$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$x^2 + x + 1$	$x^2 + x + 1$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$0x^2 - x + 0$	$0x^2 - x + 0$
$+(x^2 + 2x + 1)$	$+(x^2 + 2x + 1)$
$0x^2 + x + 1$	$0x^2 + x + 1$

Example 4 through 6

$2x^2 + 3x + 2$	$2x^2 + 3x + 2$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$x^2 + x + 1$	$x^2 + x + 1$
$-(x^2 + 2x + 1)$	$-(x^2 + 2x + 1)$
$+$	$+$
$0x^2 - x + 0$	$0x^2 - x + 0$
$+(x^2 + 2x + 1)$	$+(x^2 + 2x + 1)$
$0x^2 + x + 1$	$0x^2 + x + 1$

$$\begin{array}{r|l} 5+2-1-1+0+1 & -3 \\ 5+5-10+5+15-5 & 20 \\ \hline 0-3+9-6-15+6 & -23 \end{array}$$

Multiply 1 by $-\frac{3}{1} \rightarrow -3$:

$$-3-5+6-5-9+8 \quad -12$$

Subtract from 3

$$-3-1+2+3+1+3 \quad 16$$

$$-2-5+6-3-9+3 \quad -12$$

$$\boxed{0+2-4+6+10+6 \quad 28}$$

Multiply 1 by $\frac{4}{1} \rightarrow 4$ to have:

$$4+4-8+4+12-4 \quad 16$$

Subtract from 6

$$4+3+1-6-3-2 \quad -27$$

$$4+4-8+4+12-4 \quad 16$$

$$\boxed{0-1+9-10-15+2 \quad -43}$$

Therefore:

$1+1-2+1+3-1$	4
$0-5+5+0-5-1$	12
$0+2-1-2-1+2$	-19
$0-3+9-0-13+6$	-13
$0+2-4+6+10+0$	28
$0-1+9-10-15+2$	-43

Multiply 2 through by $-\frac{2}{3}$ to have:

$$0+2-\frac{10}{3}+0+\frac{10}{3}+\frac{2}{3} \quad -8$$

Subtract from 3'

$$0+2-1-2-1+2 \quad 19$$

$$-0+2-\frac{10}{3}+0+\frac{10}{3}+\frac{2}{3} \quad -8$$

$$\boxed{0+0+\frac{9}{3}-2-\frac{13}{3}+\frac{4}{3} \quad -11}$$

Multiply 2 through by $-\frac{5}{3} \rightarrow 1$:

$$0-3+5+0-5-1 \quad 12$$

Subtract from 4'

$$0 \ 2 + 9 - 6 - 15 + 0 \quad | \quad -23$$

$$0 \ -3 + 5 + 0 - 5 - 1 \quad | \quad 12$$

$$0 + 0 + 4 - 6 - 8 + 7 \quad | \quad -35$$

Multiply 2^1 through by $-2/3$

$$0 \ 2 - 10/3 + 0 - 10/3 + 2/3 \quad | \quad -8$$

Subtract from 2^1

$$0 \ 2 - 4 + 6 + 10 + 0 \quad | \quad 29$$

$$-0 \ 2 - 10/3 + 0 - 10/3 + 2/3 \quad | \quad -8$$

$$0 \ 0 - 2/3 + 6 + 20/3 - 2/3 \quad | \quad 26$$

Multiply 2^1 through by $-1/3 \rightarrow 1/3$

$$0 \ -1 + 5/3 + 0 - 2/3 - 1/3 \quad | \quad 4$$

Subtract from 2^1

$$0 \ -1 + 9 - 10 - 15 + 2 \quad | \quad -42$$

$$-0 \ -1 + 5/3 + 0 - 5/3 - 1/3 \quad | \quad 4$$

$$0 \ 0 + 22/3 - 10 - 40/3 + 2/3 \quad | \quad -47$$

Therefore:

$$1 + 1 - 2 + 1 + 3 - 1 \quad | \quad 4$$

$$0 - 3 + 5 + 0 - 5 - 1 \quad | \quad 12$$

$$0 + 0 + 9/3 - 2 - 10/3 + 4/3 \quad | \quad -11$$

$$0 + 0 + 4 - 6 - 8 + 7 \quad | \quad -35$$

$$0 + 0 - 2/3 + 6 + 20/3 - 2/3 \quad | \quad 26$$

$$0 + 0 + 22/3 - 10 - 40/3 + 2/3 \quad | \quad -47$$

Multiply 3^1 by $4 \times 3/4 \rightarrow 12/4$

$$0 + 0 + 4 - 24/4 - 52/4 + 16/4 \quad | \quad -100/4$$

Subtract from 4^1

$$0 + 0 + 4 - 6 - 8 + 7 \quad | \quad -35$$

$$-0 + 0 + 4 - 24/4 - 52/4 + 16/4 \quad | \quad -100/4$$

$$0 + 0 + 0 - 18/4 - 4/4 + 55/4 \quad | \quad -13/4$$

Multiply 3^1 by $-2/3 \times 3/7 \rightarrow -2/4$

$$0 + 0 - 2/3 + 2/3 + 26/21 - 8/21 \quad | \quad 22/21$$

Subtract from 5^1

$$\begin{array}{r}
 0 + 0 - \frac{2}{3} + 6 + \frac{27}{3} - \frac{2}{3} \quad | \quad \frac{96}{3} \\
 - 0 + 0 - \frac{2}{3} + \frac{4}{3} + \frac{20}{3} - \frac{9}{3} \quad | \quad \frac{20}{3} \\
 \hline
 0 + 0 + 0 + \frac{38}{3} - \frac{28}{3} - \frac{2}{3} \quad | \quad \frac{280}{3}
 \end{array}$$

Multiply 5'' by $\frac{20}{3} \times \frac{3}{2} \rightarrow \frac{20}{2}$

$$\begin{array}{r}
 0 + 0 + \frac{22}{3} - \frac{4}{3} - \frac{280}{21} + \frac{28}{21} \quad | \quad -\frac{242}{7}
 \end{array}$$

Subtract from 6''

$$\begin{array}{r}
 0 + 0 + \frac{22}{3} - 10 - \frac{40}{3} + \frac{2}{3} \quad | \quad -42 \\
 - 0 + 0 + \frac{22}{3} - \frac{40}{3} - \frac{280}{21} + \frac{28}{21} \quad | \quad -\frac{392}{7} \\
 \hline
 0 + 0 + 0 - \frac{20}{3} + \frac{2}{3} - \frac{18}{7} \quad | \quad -\frac{82}{7}
 \end{array}$$

Therefore:

$1 + 1 - 2 + 1 + 3 - 1$	4
$0 - 3 + 5 + 0 - 5 - 1$	12
$0 + 0 + \frac{9}{3} - 2 - \frac{12}{3} + \frac{4}{3}$	11
$0 + 0 + 0 - \frac{18}{3} - \frac{4}{3} + \frac{38}{3}$	$118/3$
$0 + 0 + 0 + \frac{28}{3} + \frac{28}{3} - \frac{2}{3}$	$230/3$
$0 + 0 + 0 - \frac{20}{3} + \frac{4}{3} - \frac{18}{7}$	$-82/7$

Multiply 4''' by $\frac{28}{3} \times \frac{3}{2} \rightarrow \frac{19}{1}$

$$\begin{array}{r}
 0 + 0 + 0 + \frac{38}{3} + \frac{26}{63} - \frac{204}{21} \quad | \quad \frac{2146}{63}
 \end{array}$$

Subtract from 5'''

$$\begin{array}{r}
 0 + 0 + 0 + \frac{38}{3} + \frac{26}{63} - \frac{2}{3} \quad | \quad \frac{230}{7} \\
 - 0 + 0 + 0 + \frac{38}{3} + \frac{26}{63} - \frac{204}{21} \quad | \quad \frac{2146}{63} \\
 \hline
 0 + 0 + 0 + 0 + \frac{38}{9} + \frac{29}{9} \quad | \quad -\frac{11}{9}
 \end{array}$$

Multiply 4''' by $-\frac{26}{3} \times \frac{3}{2} \rightarrow \frac{13}{1}$

$$\begin{array}{r}
 0 + 0 + 0 - \frac{26}{3} + \frac{2}{3} - \frac{18}{7} \quad | \quad -\frac{82}{7} \\
 - 0 + 0 + 0 - \frac{26}{3} - \frac{52}{63} + \frac{42}{21} \quad | \quad -\frac{1924}{63} \\
 \hline
 0 + 0 + 0 + 0 + \frac{10}{9} - \frac{26}{9} \quad | \quad \frac{98}{9}
 \end{array}$$

Therefore:

$1 + 1 - 2 + 1 + 2 - 1$	4
$0 - 3 + 5 + 0 - 5 - 1$	12
$0 + 0 + \frac{7}{3} - 2 - \frac{13}{3} + \frac{4}{3}$	-11
$0 + 0 + 0 - \frac{19}{3} - \frac{4}{3} + \frac{25}{3}$	$-\frac{112}{3}$
$0 + 0 + 0 + 0 + \frac{38}{9} + \frac{27}{9}$	$-\frac{11}{9}$
$0 + 0 + 0 + 0 + \frac{10}{9} - \frac{22}{9}$	$\frac{28}{9}$

Multiply 5^{10} by $\frac{10! \times 9!}{38} \rightarrow \frac{8}{9}$

$$0 + 0 + 0 + 0 + \frac{10}{9} + \frac{22}{9} \times \frac{8}{9} = -\frac{52}{9}$$

Subtract from 0^{10}

$0 + 0 + 0 + 0 + \frac{10}{9} - \frac{22}{9}$	$\frac{28}{9}$
$0 + 0 + 0 + 0 + \frac{10}{9} + \frac{22}{9}$	$-\frac{52}{9}$
$0 + 0 + 0 + 0 + 0 - \frac{212}{9}$	$\frac{212}{9}$

Therefore:

$1 + 1 - 2 + 1 + 2 - 1$	4
$0 - 3 + 5 + 0 - 5 - 1$	12
$0 + 0 + \frac{7}{3} - 2 - \frac{13}{3} + \frac{4}{3}$	-11
$0 + 0 + 0 - \frac{19}{3} - \frac{4}{3} + \frac{25}{3}$	$-\frac{112}{3}$
$0 + 0 + 0 + 0 + \frac{38}{9} + \frac{27}{9}$	$-\frac{11}{9}$
$0 + 0 + 0 + 0 + 0 - \frac{212}{9}$	$\frac{212}{9}$

For Back Substitution:

$$-\frac{212}{9} T_8 = \frac{212}{9}$$

$$T_8 = -1$$

$$T_8 \frac{58}{9} + T_8 \frac{27}{3} = -11$$

$$T_8 \frac{58}{9} + (-1) \frac{27}{3} = -11 \rightarrow T_8 \frac{58}{9} = -11 + \frac{27}{3}$$

$$\frac{58}{9} T_8 = \frac{26}{9}$$

$$T_8 = 2$$

$$\frac{-18T_4 - 5T_5 + 22T_6 = -113}{2 \quad 7 \quad 7 \quad 7}$$

$$\frac{-18T_4 - 5 - 22 = -113}{2 \quad 7 \quad 7 \quad 7}$$

$$\frac{-18T_4 = -113 + 5 + 22}{2 \quad 7 \quad 7 \quad 7}$$

$$\frac{-18T_4 = -72}{2 \quad 7 \quad 7 \quad 7}$$

$$T_4 = 4$$

$$\frac{2T_5 - 2T_4 - 13T_6 + 4T_7 = -11}{2 \quad 2 \quad 2 \quad 2}$$

$$\frac{2T_5 - 8 - 26 - 4 = -11}{2 \quad 2 \quad 2 \quad 2}$$

$$\frac{2T_5 = -11 + 8 + 26 + 4}{2 \quad 2 \quad 2 \quad 2}$$

$$\frac{2T_5 = 7}{2}$$

$$T_5 = 3$$

$$-3T_6 + 5T_7 + 0 - 5T_4 - T_5 = 12$$

$$-3T_6 + 15 + 0 - 10 + 1 = 12$$

$$-3T_6 = 12 - 15 + 10 - 1$$

$$-3T_6 = 6$$

$$T_6 = -2$$

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 = 4 + 2 + 6 - 4 - 6 + 1$$

$$T_1 = 1$$

$$\therefore T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 8$$

$$T_4 = 4$$

$$T_5 = 3$$

$$T_6 = -1$$