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Assignment 2.

1) $f(x) = e^{-0.5x}(4-x)-2$. Given the initial guess value of $0.5 = x_0$ maximum percentage absolute error = 1E-9. Find;
(i) find the root of the function manually and with the aid of matlab using Newton raphson Iterative method.

Solution.

Manually

From Newton Raphson Iteration.

$$x_{i+1} = x_i - \frac{f(x)}{f'(x)}$$

$f(x) = e^{-0.5x}(4-x)-2$

$f'(x) = u = e^{-0.5x}$ $\quad v = 4-x$

$\frac{du}{dx} = -0.5e^{-0.5x}$ $\quad \frac{dv}{dx} = -1$

from product rule;

$u\frac{dv}{dx} + v\frac{du}{dx}$

$-e^{-0.5x} + ((-0.5e^{-0.5x} \times 4) + x0.5e^{-0.5x})$

$-e^{-0.5x} - 2e^{-0.5x} + 0.5e^{-0.5x}$

$f'(x)\ x\ 0.5e^{-0.5x} - 3e^{-0.5x}$

$e^{-0.5x}(0.5x-3)$

Percentage absolute error $= \left[\frac{x_{i+1} - x_i}{x_{i+1}}\right] \times 100$

For 1st Iteration,

Let $x_i = 0.5$

$f(x_0) = (4-0.5)e^{-0.5(0.5)} - 2$

$f(x_0) = 0.7258027407$

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$$f'(x_0) = e^{-0.5(0.5)}\left[(0.5\times0.3)-3\right]$$
$$= -2.141702158$$

$$x_{i+1} = 0.5 - \frac{0.7258027407}{-2.141702153}$$

$$x_{i+1} = 0.838890606$$

$$\%\text{ absolute error} = \left[\frac{0.838890606 - 0.5}{0.838890606}\right] \times 100\%$$

$$\%\text{ absolute error} = 40.39747298\ \%$$

For 2nd Iteration.

$$x_i = 0.838890606$$
$$f(x_i) = (4 - 0.838890606)e^{-0.5(0.838890606)} - 2$$
$$f(x_i) = 0.078149297 94.$$
$$f'(x_i) = e^{-0.3\times(0.838890606)}\left[(0.5\times0.838890606)-3\right]$$
$$f'(x_i) = -1.896486032.$$

$$x_{i+1} = 0.838890606 - \left[\frac{0.07814929794}{-1.696486032}\right]$$

$$x_{i+1} = 0.8849560003$$
$$\%\text{ absolute error} = \left[\frac{0.8849560003 - 0.838890606}{0.8849560003}\right] \times 100$$

$$\%\text{ absolute error} = 5.205388094\ \%$$

For 3rd Iteration.

$$x_i = 0.8849560003.$$
$$f(x_2) = (4 - 0.8849560003)e^{-0.5(0.8849560003)}$$
$$f(x_2) = 0.0012365 7519.$$
$$f'(x_2) = e^{(-0.5\times0.8849560003)}\left[(0.5\times0.8849560003)-3\right]$$
$$f'(x_2) = -1.643060762$$

$$x_{i+1} = 0.8849560003 - \left[\frac{0.00123657319}{-1.643060782}\right]$$

$$x_{i+1} = 0.885708605$$

$$\% \text{ absolute error} = \left[\frac{0.885708605 - 0.8849560003}{0.885708605}\right] \times 100$$

$$\% \text{ absolute error} = 0.08497203912 \%$$

For 4th Iteration.

$$\text{Let } x_i = 0.885708605$$

$$f(x_3) = (4 - 0.885708605)\, e^{-0.3(0.885708605)} - 2$$

$$f(x_3) = 3.23521411 \times 10^{-7}$$

$$f'(x_3) = e^{(-0.5 \times 0.885708605)}\left[(0.5 \times 0.885708605) - 3\right]$$

$$f'(x_3) = -1.642200929$$

$$x_{i+1} = 0.885708605 - \left[\frac{3.23521411 \times 10^{-7}}{-1.642200929}\right]$$

$$x_{i+1} = 0.885708802$$

$$\% \text{ absolute error} = \left[\frac{0.885708802 - 0.885708605}{0.885708802}\right] \times 100\%$$

$$\% \text{ absolute error} = 2.224261137 \times 10^{-5} \%$$

For 5th Iteration.

$$x_4 = 0.885708802$$

$$f(x_4) = (4 - 0.885708802)\, e^{-0.5(0.885708802)} - 2$$

$$f(x_4) = 7.851 \times 10^{-12}$$

$$f'(x_4) = e^{(-0.5 \times 0.885708802)}\left[(0.5 \times 0.885708802) - 3\right]$$

$$f'(x_4) = -1.642200704$$

$$x_{i+1} = 0.885708802 - \left[\frac{7.831 \times 10^{-12}}{-1.642200704}\right]$$

$$x_{i+1} = 0.885708802.$$

Iterations	$x_{(i+1)}$	% error
1	0.838890606	40.39747299
2	0.8849560003	5.205388094
3	0.885708605	0.0849720391 2
4	0.885708802	$2.2242611 37 \times 10^{-3}$
5	0.885708802	0

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