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Assignment 3

1) In the model of a system having thermo coupled measuring temp $T(^{\circ}C)$, at its different points is given by the set of expressions in eqt (1). Estimate the value of the temperature using:

a) Gauss elimination method manually;

$$\begin{cases} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 + T_4 - 6T_5 - 3T_6 = -27 \end{cases}$$

Solution

forward Elimination (eliminating T_1):

$$\begin{array}{l} \text{row 1} \rightarrow \\ \text{row 2} \rightarrow \\ \text{row 3} \rightarrow \\ \text{row 4} \rightarrow \\ \text{row 5} \rightarrow \\ \text{row 6} \rightarrow \end{array} \left[\begin{array}{cccccc} 1 & 1 & -2 & 1 & 3 & -1 \\ -2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{array} \right] \begin{array}{l} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{array}$$

multiply row 1 by 2: $2 [1 \ 1 \ -2 \ 1 \ 3 \ -1 \ 4]$
 $= [2 \ 2 \ -4 \ 2 \ 6 \ -2 \ 8]$

Subtract from row 2:

$$\begin{array}{r} [2 \ 2 \ -4 \ 2 \ 6 \ -2 \ 8] \\ - [2 \ -1 \ 1 \ 2 \ 1 \ -3 \ 20] \\ \hline = [3 \ -5 \ 0 \ 5 \ 1 \ -12] \rightarrow \text{new row 1:} \end{array}$$

$\Rightarrow 3T_2 - 5T_3 + 0T_4 + 5T_5 + T_6 = -12 \dots (1)$

Subtract row 3 from row 1

$$\begin{array}{r} [1 \ 1 \ -2 \ 1 \ 3 \ -1 \ | \ 4] \\ - [1 \ 3 \ -3 \ -1 \ 2 \ 1 \ | \ -15] \end{array}$$

$$\Rightarrow [0 \ -2 \ 1 \ 2 \ 1 \ -2 \ | \ 19] \text{ [new row 2]}$$

$$= -2T_2 + T_3 + 2T_4 + T_5 - 2T_6 = 19 \text{ --- (ii)}$$

multiply row 1 by 3.

$$\begin{array}{l} 3 \times [1 \ 1 \ -2 \ 1 \ 3 \ -1 \ | \ 4] \\ = [3 \ 3 \ -6 \ 3 \ 9 \ -3 \ | \ 12] \end{array}$$

Subtract from row 4

$$= [3 \ 3 \ -6 \ 3 \ 9 \ -3 \ | \ 12] - [5 \ 2 \ -1 \ -1 \ 2 \ 1 \ | \ -5]$$

$$\Rightarrow 3T_2 - 9T_3 + 6T_4 + 13T_5 - 6T_6 = 23 \text{ --- (iii)}$$

multiply row 1 by 3.

$$\begin{array}{l} 3 \times [1 \ 1 \ -2 \ 1 \ 3 \ -1 \ | \ 4] \\ = [3 \ 3 \ -6 \ 3 \ 9 \ -3 \ | \ 12] \end{array}$$

Add to row 3

$$\Rightarrow [3 \ 3 \ -6 \ 3 \ 9 \ -3 \ | \ 12] + [-3 \ -1 \ 2 \ 3 \ 1 \ 3 \ | \ 16]$$

$$\Rightarrow [0 \ -2 \ -4 \ 6 \ 10 \ 0 \ | \ 28] \text{ (new row 4)}$$

$$\Rightarrow 2T_2 - 4T_3 + 6T_4 + 10T_5 + 0T_6 = 28 \text{ --- (iv)}$$

Multiply row 1 by 4

$$\begin{array}{l} 4 \times [1 \ 1 \ -2 \ 1 \ 3 \ -1 \ | \ 4] \\ \Rightarrow [4 \ 4 \ -8 \ 4 \ 12 \ -4 \ | \ 16] \end{array}$$

Subtract from row 6.

$$[4 \ 4 \ -8 \ 4 \ 12 \ -4 \ | \ 16] - [4 \ 3 \ 1 \ -6 \ -3 \ -2 \ | \ -29]$$

$$\frac{1}{2} T_2 - 9T_3 + 10T_4$$

$$[0 \ 1 \ -1 \ 10 \ 15 \ -2 \ | \ 48] \text{ (new row 5)}$$

$$\Rightarrow T_2 - 9T_3 + 10T_4 + 15T_5 - 2T_6 = 48 \text{ --- (v)}$$

combining eqt (1-5); $3T_2 - 5T_3 + 0T_4 + 5T_5 + T_6 = -12$ --- (1)

$-2T_2 + T_3 + 2T_4 + T_5 - 2T_6 = 19$ --- (2)

$3T_2 - 9T_3 + 6T_4 + 13T_5 - 6T_6 = 23$ --- (3)

$2T_2 - 4T_3 + 6T_4 + 10T_5 + 0T_6 = 28$ --- (4)

$T_2 - 9T_3 + 10T_4 + 15T_5 - 2T_6 = 43$ --- (5)

forward elimination (eliminating T_2)

$$\begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \\ \text{row 4} \\ \text{row 5} \end{array} \left[\begin{array}{ccccc|c} 3 & -5 & 0 & 5 & 1 & -12 \\ -2 & 1 & 2 & 1 & -2 & 19 \\ 3 & -9 & 6 & 13 & -6 & 23 \\ 2 & -4 & 6 & 10 & 0 & 28 \\ 1 & -9 & 10 & 15 & -2 & 43 \end{array} \right]$$

multiply row 1 by $\frac{2}{3}$

$$\frac{2}{3} [3 \quad -5 \quad 0 \quad 5 \quad 1 \quad | \quad -12] = \left[2 \quad \frac{-10}{3} \quad 0 \quad \frac{10}{3} \quad \frac{2}{3} \quad | \quad -8 \right]$$

Add to row 2

$$\left[2 \quad \frac{-10}{3} \quad 0 \quad \frac{10}{3} \quad \frac{2}{3} \quad | \quad -8 \right] + \left[-2 \quad 1 \quad 2 \quad 1 \quad -2 \quad | \quad 19 \right]$$

$$= \left[0 \quad \frac{-7}{3} \quad 2 \quad \frac{13}{3} \quad \frac{-4}{3} \quad | \quad 11 \right] \text{ (new row 1)}$$

$$= \frac{-7}{3}T_3 + 2T_4 + \frac{13}{3}T_5 - \frac{4}{3}T_6 = 11 \quad \text{--- (6)}$$

Subtract row 3 from 1

$$\left[3 \quad -9 \quad 6 \quad 13 \quad -6 \quad | \quad 23 \right] - \left[3 \quad -5 \quad 0 \quad 5 \quad 1 \quad | \quad -12 \right]$$

$$\Rightarrow \left[0 \quad -4 \quad 6 \quad 8 \quad -7 \quad | \quad 35 \right] \text{ (new row 2)}$$

$$= -4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \quad \text{--- (7)}$$

multiply row 1 by $\frac{2}{3}$

$$\frac{2}{3} [3 \quad -5 \quad 0 \quad 3 \quad 1 \quad | \quad -12] = \left[2 \quad \frac{-10}{3} \quad 0 \quad \frac{10}{3} \quad \frac{2}{3} \quad | \quad -8 \right]$$

Subtract from row (4)

$$\left[2 \quad -4 \quad 6 \quad 10 \quad 0 \quad | \quad 28 \right] - \left[2 \quad \frac{-10}{3} \quad 0 \quad \frac{10}{3} \quad \frac{2}{3} \quad | \quad -8 \right]$$

$$= \left[0 \quad \frac{-2}{3} \quad 6 \quad \frac{20}{3} \quad \frac{-2}{3} \quad | \quad 36 \right] \text{ (new row 3)}$$

$$\Rightarrow \frac{-2}{3}T_3 + 6T_4 + \frac{20}{3}T_5 - \frac{2}{3}T_6 = 36 \quad \text{--- (8)}$$

multiply row 1 by $\frac{1}{3}$

$$\frac{1}{3} \times [3 \ -5 \ 0 \ 5 \ 1 \ | \ -12] = [1 \ -\frac{5}{3} \ 0 \ \frac{5}{3} \ \frac{1}{3} \ | \ -4]$$

Subtract from row 5.

$$[1 \ -9 \ 10 \ 15 \ -2 \ | \ 43] - [1 \ -\frac{5}{3} \ 0 \ \frac{5}{3} \ \frac{1}{3} \ | \ -4]$$
$$= [-\frac{22}{3} \ 10 \ \frac{40}{3} \ -\frac{7}{3} \ | \ 47] \text{ (new row 4)}$$
$$= \frac{22}{3}T_3 + 10T_4 + \frac{40}{3}T_5 - \frac{7}{3}T_6 = 47 \text{ --- (9)}$$

Combining eqt (6-9).

$$\frac{9}{3}T_3 + 2T_4 + \frac{15}{3}T_5 - \frac{4}{3}T_6 = 11 \text{ --- (6)}$$

$$-4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \text{ --- (7)}$$

$$-\frac{2}{3}T_3 + 6T_4 + \frac{20}{3}T_5 + \frac{7}{3}T_6 = 36 \text{ --- (8)}$$

$$-\frac{22}{3}T_3 + 10T_4 + \frac{40}{3}T_5 - \frac{7}{3}T_6 = 47 \text{ --- (9)}$$

Forward elimination (eliminating T_3).

row 1 \rightarrow

row 2 \rightarrow

row 3 \rightarrow

row 4 \rightarrow

Multiply row 1 by $\frac{12}{7}$

$$\frac{12}{7} [-\frac{7}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ | \ 11] = [-4 \ \frac{24}{7} \ \frac{52}{7} \ -\frac{16}{7} \ | \ \frac{132}{7}]$$

Eq 2 to be subtracted from this.

$$[-4 \ \frac{24}{7} \ \frac{52}{7} \ -\frac{16}{7} \ | \ \frac{132}{7}] - [-4 \ 6 \ 8 \ -7 \ | \ 35]$$

$$\text{New row 1} [0 \ -\frac{15}{7} \ -\frac{4}{7} \ \frac{33}{7} \ | \ -\frac{113}{7}]$$

$$\Rightarrow -\frac{15}{7}T_4 - \frac{4}{7}T_5 + \frac{33}{7}T_6 = -\frac{113}{7} \text{ --- (10)}$$

Multiply row 1 by $\frac{7}{2}$

$$\frac{7}{2} [-\frac{7}{3} \ 2 \ \frac{13}{3} \ -\frac{4}{3} \ | \ 11] = [-\frac{7}{3} \ 4 \ \frac{26}{3} \ -\frac{8}{3} \ | \ \frac{22}{3}]$$

Subtract row 3 from above;

$$[-\frac{7}{3} \ 4 \ \frac{26}{3} \ -\frac{8}{3} \ | \ \frac{22}{3}] - [-\frac{7}{3} \ 6 \ \frac{20}{3} \ -\frac{2}{3} \ | \ 36]$$

$$= [0 \ -\frac{38}{3} \ -\frac{38}{3} \ \frac{2}{3} \ | \ -\frac{230}{3}] \text{ (new row 2)}$$

$$= -\frac{38}{3}T_4 - \frac{38}{3}T_5 + \frac{2}{3}T_6 = -\frac{230}{3} \text{ --- (11)}$$

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Assignment 3 (Continuation)

Multiply row 1 by $\frac{22}{7}$

$$\frac{22}{7} \begin{bmatrix} -7/3 & 2 & 13/3 & -4/3 & | & 11 \end{bmatrix} = \begin{bmatrix} -22/3 & 44/7 & 286/21 & -88/21 & | & 242/7 \end{bmatrix}$$

$$\begin{bmatrix} -22/3 & 44/7 & 286/21 & -88/21 & | & 242/7 \end{bmatrix} - \begin{bmatrix} -22/3 & 10 & 40/3 & -7/3 & | & 47 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -26/7 & 2/7 & -13/7 & | & -87/7 \end{bmatrix} \text{ (New row 3)}$$

$$-26/7 T_4 + 2/7 T_5 - 13/7 T_6 = -87/7 \text{ --- (12)}$$

Combining eqt (10-12)

$$-18/7 T_4 - 4/7 T_5 + 33/7 T_6 = -113/7 \text{ --- (10)}$$

$$-38/7 T_4 - 380/7 T_5 + 2/7 T_6 = -230/7 \text{ --- (11)}$$

$$-26/7 T_4 + 2/7 T_5 - 13/7 T_6 = -87/7 \text{ --- (12)}$$

Forward elimination (eliminating T_4)

$$\begin{bmatrix} -18/7 & -4/7 & 33/7 & | & -113/7 \\ -38/7 & -38/7 & 2/7 & | & -230/7 \\ -26/7 & 2/7 & -13/7 & | & -87/7 \end{bmatrix} \begin{matrix} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{matrix}$$

Multiply row 1 by $\frac{19}{9}$

$$\frac{19}{9} \times \begin{bmatrix} -18/7 & -4/7 & 33/7 & | & -113/7 \end{bmatrix} = \begin{bmatrix} -38/7 & -76/63 & 209/21 & | & -2147/63 \end{bmatrix}$$

Subtract row 2 from the above.

$$\begin{bmatrix} -38/7 & -76/63 & 209/21 & | & -2147/63 \end{bmatrix} - \begin{bmatrix} -38/7 & -38/7 & 2/7 & | & -230/7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 38/9 & 29/3 & | & -11/9 \end{bmatrix}$$

$$= 38/9 T_5 + 29/3 T_6 = -11/9 \text{ --- (13)}$$

Multiply row 1 by $\frac{13}{9}$

$$\frac{13}{9} \times \begin{bmatrix} -18/7 & -4/7 & 33/7 & | & -113/7 \end{bmatrix} = \begin{bmatrix} -26/7 & -52/63 & 143/21 & | & -1469/63 \end{bmatrix}$$

Subtract row 3 from the above;

$$\left[\begin{array}{ccc|c} -26/7 & -52/63 & 142/21 & -1169/63 \end{array} \right] - \left[\begin{array}{ccc|c} 26/7 & 2/7 & -13/7 & -87/7 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 0 & -10/9 & 26/3 & -98/9 \end{array} \right]$$

$$\therefore -10/9 T_5 + 26/3 T_6 = -98/9 \quad \dots (14)$$

Combining eqs (13 and 14)

$$38/9 T_5 + 24/3 T_6 = -11/9 \quad \dots (13)$$

$$-10/9 T_5 + 26/3 T_6 = -98/9 \quad \dots (14)$$

From eq (13) make T_6 subject of formula

$$T_6 = \left(\frac{-11}{9} - \frac{38}{9} T_5 \right) \times \frac{3}{24} \quad \dots (15)$$

Backward Substitution.

Substitute T_6 in eq (14)

$$-10/9 T_5 + 26/3 \left(\frac{-11}{9} - \frac{38 T_5}{9} \right) \frac{3}{24} = -98/9$$

$$-10/9 T_5 \left(\frac{-286}{27} - \frac{988 T_5}{27} \right) \frac{3}{24} = -98/9$$

$$-10/9 T_5 - \frac{286}{27} - \frac{988}{27} T_5 = -98/9$$

$$-142/27 T_5 = -284/27$$

$$T_5 = -284/142 = 2$$

Substitute T_5 in eq (15); $T_6 = \left(\frac{-11}{9} - \frac{38 \times 2}{9} \right) \times \frac{3}{24}$

$$\left(\frac{-11}{9} - \frac{76}{9} \right) \times \frac{3}{24}$$

$$T_6 = -1$$

Substitute T_5 and T_6 in eq (10)

$$-18/7 T_4 - 8/7 - 33/7 = -113/7$$

$$-18/7 T_4 = -113/7 + 33/7 + 8/7$$

$$-18/7 T_4 = -72/7$$

$$T_4 = -72/7 \times -7/18$$

$$T_4 = 4$$

Backward Substitution.

Substitute T_4 , T_5 and T_6 in eqt (7)
from eqt (7)

$$-4T_3 + 6T_4 + 8T_5 - 7T_6 = 35 \quad \dots (7)$$

where; $T_4 = 4$

$$T_5 = 2$$

$$T_6 = -1$$

$$-4T_3 + 6(4) + 8(2) - 7(-1) = 35$$

$$-4T_3 + 24 + 16 + 7 = 35$$

$$-4T_3 = 35 - 47$$

$$-4T_3 = -12$$

$$T_3 = 3$$

from eqt (1)

$$3T_2 - 5T_3 + 5T_5 + T_6 = -12 \quad \dots (1)$$

Substitute T_3 , T_5 and T_6 in eqt (1)

$$3T_2 - 5(3) + 5(2) - 1 = -12$$

$$3T_2 = -12 + 15 - 10 + 1$$

$$3T_2 = -6$$

$$T_2 = -2$$

from the initial equations.

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

Substitute T_2 , T_3 , T_4 , T_5 and T_6

$$T_1 + (-2) - 2(3) + 4 + 3(2) + 1 = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 = 4 + 2 + 6 - 4 = 6 - 1$$

$$T_1 = 1$$

$$T_6 = -1$$

$$T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$