31/01/19 FIDE-AKWUOBI ANTHONY CHIZALU 17/ENG06/037 MECHANICAL ENGINEERING ENG 252 200 LVL

1 (a) A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivative of y with respect to x.

Examples: (i) $\frac{dy}{dx} = 2 + \frac{y}{x}$

(ii) $\frac{dy}{dx} = y + \frac{y}{x}$

(b) $y = Ae^{-4x} + Be^{-6x}$

(i) A second order differential equation.

(ii) A second order differential equation can be formed because it contains 2 constants in the degenerate equation.

(iii) $y = Ae^{-4x} + Be^{-6x}$

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{—(1)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{—(2)}$$

Solving eqn (1) and (2) simultaneously:

Multiply eqn (1) by 6

$$\therefore 6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{—(3)}$$

$$+$$

$$\frac{d^2y}{dx^2} = +16Ae^{-4x} + 36Be^{-6x} \quad \text{—(4)}$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \quad \text{—(5)}$$

Substituting eqn-(5) into eqn (1)

$$\frac{dy}{dx} = A\left(\frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

Multiply through by 2

$$\therefore 2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2\frac{dy}{dx} - 6\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B \qquad \therefore \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equation

$$\therefore y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8\cancel{e^{-4x}}} \times \cancel{e^{-4x}} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12\cancel{e^{-6x}}} \times \cancel{e^{-6x}}$$

$$y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12}$$

$$y = \frac{-72\frac{dy}{dx} - 12\frac{d^2y}{dx^2} + 32\frac{dy}{dx} + 8\frac{d^2y}{dx^2}}{96}$$

$$y = \frac{-40\frac{dy}{dx} - 4\frac{d^2y}{dx^2}}{96}$$

$$96y = -40\frac{dy}{dx} - 4\frac{d^2y}{dx^2}$$

$$24y = -10\frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\therefore \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$