02/03/19 FIDE-AKWUOBI ANTHONY CHIZALU 17/ENG06/037 MECHANICAL ENGINEERING 200 LVL ENG 282

(1) Given that $F = x^2 i + (3x+2)j + \sin x k$; Find (a) $\frac{dF}{dx}$ (b) $\frac{d^2F}{dx^2}$ (c) $\left|\frac{dF}{dx}\right|$ (d) $\frac{d}{dx}(F \cdot F)$ at $x=1$.

Solution

$$F = x^2 i + (3x+2)j + \sin x k$$

(a) $\frac{dF}{dx} = 2xi + (3+0)j + \cos x k$

$$\therefore \frac{dF}{dx} = 2xi + 3j + \cos x k$$

(b) $\frac{d^2F}{dx^2} = 2i - \sin x k$

(c) $\left|\frac{dF}{dx}\right| = \sqrt{(2x)^2 + (3)^2 + (\cos x)^2}$

$$= \sqrt{4x^2 + 9 + \cos^2 x}$$

At $x=1 \rightarrow \sqrt{4(1)^2 + 9 + \cos^2(1)}$

$$= \sqrt{4 + 9 + 0.999} = \sqrt{13.999} = 3.74$$

(d) $\frac{d}{dx}(F \cdot F) \rightarrow (F \cdot F) = [(x^2 i) + (3x+2)j + \sin x k] \cdot [(x^2)i + (3x+2)j + \sin x k]$

$$= x^4 + (9x^2 + 6x + 6x + 4) + \sin^2 x$$

$$= x^4 + 9x^2 + 12x + 4 + \sin^2 x$$

$$\therefore \frac{d}{dx}(F \cdot F) = 4x^3 + 18x + 12 + 2\sin x \cos x$$

$\frac{d}{dx}(F \cdot F)$ at $x=1$ $\rightarrow 4(1)^3 + 18(1) + 12 + 2\sin(1)\cos(1)$

$$= 4 + 18 + 12 + 2(0.0175)(0.999)$$

$$= 4 + 18 + 12 + 0.035$$

$$= 34.035.$$

(2) If $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k$. Determine (a) $\frac{dr}{dt}$ (b) $\frac{d^2r}{dt^2}$ (c) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

Solution

(a) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t j + 6e^{2t}k$

(b) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$

(c) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$ $\therefore$ $2i + [18\sin(3)(0)]j + 12e^{2(0)}k$

$= 2i + 18\sin 0 j + 12e^0 k$

$= 2i + 18(0)j + 12(1)k$

$= 2i + 12k$

$\therefore \left|\frac{d^2r}{dt^2}\right| = \sqrt{(2)^2 + (12)^2}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{4 + 144} = \sqrt{148} = 12.17$