

Assignment IV

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MECHANICAL ENGINEERING

ENR 322: ENGINEERING MATHS IV

Question:

a.) Using an initial guess vector of $m_0 = [0; 0; 0]$, determine the values of the variables in Eqn (1) using Jacobi iterative method manually showing only 3 iterations.

b.) Write a MATLAB mfile program to solve the problem in (a) and tabulate the results showing the no of iterations, the corresponding values of the variables and the error, which is calculated from the norms. Take the tolerance of the error to be 10^{-15} .

$$\begin{cases} 10m_1 - 2m_2 + m_3 = 9 \\ -2m_1 + 10m_2 - 2m_3 = 12 \\ -2m_1 - 5m_2 + 10m_3 = 18 \end{cases} \quad \text{--- (1)}$$

Solution

a) using Jacobi iterative method

making m_1, m_2 & m_3 in eqn 1-3

$$m_1 = \frac{2m_2 - m_3 + 9}{10} = 0.2m_2 - 0.1m_3 + 0.9$$

$$m_2 = \frac{2m_1 + 2m_3 + 12}{10} = 0.2m_1 + 0.2m_3 + 1.2$$

$$m_3 = \frac{2m_1 + 5m_2 + 18}{10} = 0.2m_1 + 0.5m_2 + 1.8$$

Matrix notation $Ax = b$

$$\begin{bmatrix} m_1^{(k)} \\ m_2^{(k)} \\ m_3^{(k)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} m_1^{(k-1)} \\ m_2^{(k-1)} \\ m_3^{(k-1)} \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

Initial : $m_0 = [0; 0; 0]$

$$m^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

First iteration:

$$\begin{bmatrix} m_1^{(1)} \\ m_2^{(1)} \\ m_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

$$m_1^{(1)} = \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

Second iteration:

$$\begin{bmatrix} m_1^{(2)} \\ m_2^{(2)} \\ m_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$
$$= \begin{bmatrix} 0 + 0.24 + (-0.18) \\ 0.18 + 0 + 0.36 \\ 0.18 + 0.6 + 0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.96 \\ 1.74 \\ 2.58 \end{bmatrix}$$

$$M^{(2)} = \begin{bmatrix} 0.96 \\ 1.74 \\ 2.58 \end{bmatrix}$$

Third iteration:

$$\begin{bmatrix} m_1^{(3)} \\ m_2^{(3)} \\ m_3^{(3)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.96 \\ 1.74 \\ 2.58 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$
$$= \begin{bmatrix} 0 + 0.348 + (-0.258) \\ 0.192 + 0 + 0.516 \\ 0.192 + 0.87 + 0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.99 \\ 1.91 \\ 2.86 \end{bmatrix}$$

$$M^{(3)} = \begin{bmatrix} 0.99 \\ 1.91 \\ 2.86 \end{bmatrix}$$

b) MATLAB PROGRAM CODE

1 function X = myjacobi(A, b, x0)

2 A = [10 -2 1; -2 10 -2; -2 -5 10];

3 b = [9 12 18]';

4 x0 = [0 0 0]';

5 max1 = 100

6 tol = 0.00000000000000000001;

7 n = length(b);

8 for k = 1:max1

9 for i = 1:n

10 X(i) = (b(i) - A(i, [1:i-1, i+1:n]) * x0([1:i-1, i+1:n])) / A(i, i)

11 end

12 err = abs(norm(X' - x0));

13 relerr = err / (norm(X));

14 x0 = X';

15 if (err < tol) | (relerr < tol)

16 break

17 end

18 end

19 X = X'

20 table = [i' X' relerr']