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1(a) Differential equation

Differential equation is a relationship between an independent variable x and dependent variable y and one or more derivative of y with respect to x.

Examples

(i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2$

(ii) $x\frac{dy}{dx} = y^2 + 1$

(b)

(iii) $y = Ae^{-4x} + Be^{-6x}$

$\frac{dy}{dx} = -4Ae^{-4x} + (-6Be^{-6x}$

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$ --- (1)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ .... (2)

Multiply eqn (1) by 6

$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$ .... (3)

Adding eqn (3) and eqn 2

$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$

$+ \frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$

$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$ ... (4)

$A = -\frac{1}{8e^{-4x}}\left(6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right)$ ... (5)

put eqn (5) into eqn (2)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$

$\frac{d^2y}{dx^2} = 16\left[-\frac{1}{8e^{-4x}}\left(6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right)\right] + 36Be^{-6x}$

$\frac{d^2y}{dx^2} = -2\left(6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) + 36Be^{-6x}$

$\frac{d^2y}{dx^2} = -12\frac{dy}{dx} + \left(-2\frac{d^2y}{dx^2}\right) + 36Be^{-6x}$

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$$\frac{d^2y}{dx^2} = -12\frac{dy}{dx} - 2\frac{d^2y}{dx^2} + 36Be^{-6x}$$

$$36Be^{-6x} = 2\frac{d^2y}{dx} + \frac{d^2y}{dx^2} + 12\frac{dy}{dx}$$

$$36Be^{-6x} = 3\frac{d^2y}{dx} + 12\frac{dy}{dx}$$

Divide through by 3

$$12Be^{-6x} = \frac{d^2y}{dx} + 4\frac{dy}{dx}$$

$$B = \left(\frac{d^2y}{dx} + 4\frac{dy}{dx}\right) \times \frac{1}{12Be^{-6x}}$$

$$y = Ae^{-4x} + Be^{-6x}$$

Substituting A and B into the above equation

$$y = e^{-4x} \times \frac{1}{8e^{-4x}}\left(6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) + e^{-6x} \times \frac{1}{12e^{-6x}}\left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right)$$

$$y = -\frac{1}{8}\left(6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) + \frac{1}{12}\left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right)$$

$$y = \frac{-3\left(6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) + 2\left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right)}{24}$$

Cross multiply

$$24y = -18\frac{dy}{dx} - 3\frac{d^2y}{dx^2} + 2\frac{d^2y}{dx^2} + 8\frac{dy}{dx}$$

$$24y = -10\frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0 \quad \cdots (6)$$

∴ Equation (6), is the required equation

(i) The equation is a 2nd order differential equation

(ii) The highest power of the derivative is 2

(iii) The differential equation has two arbitrary constant (A and B)

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