Arenu Fanny Babajide
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Mechatronics Engineering

1.) A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivative of y with respect to x

Example i) $dy/dx = 2 + \frac{y}{x}$.

ii) $dy/dx = y + y/x$.

b.) $y = Ae^{-4x} + Be^{-6x}$.

i) A second order differential equation

ii) A second order differential equation can be formed because it contain 2 constants in the degenerate equation.

iii) $y = Ae^{-4x} + Be^{-6x}$.

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}.$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}.$$

Solving eqn (1) and (2) simultaneously

multiply eqn (1) by 6

$$6\,dy/dx = -24Ae^{-4x} - 36Be^{-6x}$$

$$-\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$\frac{6dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}.$$

$$\therefore A = \frac{6dy/dx + d^2y/dx^2}{-8e^{-4x}}$$

Substituting eqn(s) into eqn (1).

$$\frac{dy}{dx} = 4\left(\frac{4dy/dx + d^2y/dx^2}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{4dy/dx + d^2y/dx^2}{2} - 6Be^{-6x}$$

now multiply by 2.

$$2\frac{dy}{dx} = 4\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6}$$

multiply through by 2.

$$2\frac{dy}{dx} = 4\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}.$$

$$2\frac{dy}{dx} - 4\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}.$$

$$-2\frac{dy}{dx} - 4\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$\frac{-2dy/dx - d^2y/dx^2}{-12e^{-6x}} = B$$

$$B = \frac{4dy/dx + d^2y/dx^2}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equation

$$y = \frac{4dy/dx + d^2y/dx^2}{8e^{-4x}} xe^{-4x} + \frac{4dy/dx + d^2y/dx^2}{12e^{-6x}} xe^{-6x}$$

$$y = \frac{4dy/dx + d^2y/dx^2}{-8} + \frac{4dy/dx + d^2y/dx^2}{+12}$$

$$y = \frac{-72\,dy/dx - 12\,d^2y/dx^2 + 32\,dy/dx + 8\,d^2y/d}{96}$$

$$y = \frac{-40\,dy/dx - 4\,d^2y/dx^2}{96}$$

$$96y = -40\,dy/dx - 4d^2y/dx^2$$

$$24y = -10\,dy/dx - d^2y/dx^2$$

$$\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$