Dunmaye Akeem O.
Civil Engineering
17/Eng03106

1) Define a differential equation and give two examples

A differential equation is a ~~relation~~ relationship between an independent variable, a dependent variable, Y, and one or more derivative of y with respect to x

Example :

i) $x y \frac{dy^2}{dx^2} + d^2 y \frac{dy}{dx} + e^{3x} = 0$

ii) $4 \frac{d^2y}{dx^2} + \frac{dy}{dx} + 10y = \sin 2x$

b) An expression has been obtained for an engineering system to be given in equation (i)

$Y = Ae^{-4x} + Be^{-6x}$ ---(i)

i) What is the order of the differential equation that can be formed from the expression = Second Order equation

ii) Give a reason for your answer in b(i)

It is because with 1 - arbitrary constant gives a 2nd order equation

iii) Form a differential equation from the expression

$Y = Ae^{-4x} + Be^{-6x}$ ---(i)

$\frac{dy}{dx} = -4Ae^{-4x} + -6Be^{-6x}$ ---(ii)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ ---(iii)

Differentiate (ii)

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$$

$$4Ae^{-4x} = -\frac{dy}{dx} - 6Be^{-6x}$$

$$A = \left(-\frac{dy}{dx} - 6Be^{-6x}\right) \cdot \frac{1}{4e^{-4x}} \quad \text{---(iv)}$$

substitute eqn (iv) into eqn (iii)

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36 + Be^{-6x}$$

$$\frac{d^2y}{dx^2} = \left(-\frac{dy}{dx} - 6Be^{-6x}\right) e^{-4x} \cdot \frac{1}{4e^{-4x}} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} - 24Be^{-6x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} + 12Be^{-6x}$$

$$B = \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right) \cdot \frac{1}{12e^{-6x}} \quad \text{---(v)}$$

substitute eqn (v) into eqn (iv)

$$A = \left(-\frac{dy}{dx} - 6\left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right) e^{-6x} \cdot \frac{1}{12e^{-6x}}\right) \frac{1}{4e^{-4x}}$$

$$A = \left(-\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2} - 2\frac{dy}{dx}\right) \cdot \frac{1}{4e^{-4x}}$$

$$u = \left(-3\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right)\cdot\frac{1}{4e^{-4x}} \quad \text{---(ii)}$$

Put eqn (ii) and eqn (iii) in eqn (i)

$$y = Ae^{-4x} + Be^{6x} \quad \text{---(i)}$$

$$y = \left(-3\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right)e^{-4x}\cdot\frac{1}{4e^{-4x}} + \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right)e^{-6x}\cdot\frac{1}{12e^{-6x}}$$

$$y = -\frac{3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2} + \frac{1}{12}\frac{d^2y}{dx^2} + \frac{1}{3}\frac{dy}{dx}$$

$$y = -\frac{5}{12}\frac{dy}{dx} - \frac{1}{24}\frac{d^2y}{dx^2}$$

$$y = \left(-5\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right)\cdot\frac{1}{12}$$