

Maths ass1

$$y = Ae^{-4x} + Be^{-6x}$$

$$y = e^{-4x} x - \frac{1}{8e^{-4x}} \left(6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right) + e^{-6x} x \frac{1}{12e^{-6x}} \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right)$$

$$y = -\frac{1}{8} \left(6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right) + \frac{1}{12} \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right)$$

$$\underbrace{-3 \left(6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right) + 2 \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right)}_{24}$$

$$24y = -18 \frac{dy}{dx} + 3 \frac{d^2y}{dx^2} + 2 \frac{d^2y}{dx^2} + 8 \frac{dy}{dx}$$

$$24y = -10 \frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\left(\frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0 \right) \text{ 2nd order}$$

1) A differential equation is a relationship between an independent variable (x) and a dependent variable (y) and one or more derivatives of y with respect to x. Example (i) $\frac{dy}{dx} = 2 + 3/x$
ii) $\frac{dy}{dx} = y + 3/x$

a) $y = Ae^{-4x} + Be^{-6x}$

i) A second order differential equation

ii) A second order differential equation can be formed because it contains two constants in the degenerate equation

iii) $y = Ae^{-4x} + Be^{-6x}$

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$ — (1)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ — (2)

Multiply eqn 1 by 6

$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$ — (3)

add eqn 2 to 3

$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$

$A = \left(6\frac{dy}{dx} + \frac{d^2y}{dx^2} \right) \cdot \frac{1}{8Ae^{-4x}}$

Sub A in eqn 2

$\frac{d^2y}{dx^2} = 16 \left(6\frac{dy}{dx} + \frac{d^2y}{dx^2} \right) e^{-4x} \cdot \frac{1}{8e^{-4x}} + 36Be^{-6x}$

$\frac{d^2y}{dx^2} = -12\frac{dy}{dx} - 9\frac{d^2y}{dx^2} + 36Be^{-6x}$

$56Be^{-6x} = 3\frac{d^2y}{dx^2} + 12\frac{dy}{dx}$ divide through by 5

$12Be^{-6x} = \frac{d^2y}{dx^2} + 4\frac{dy}{dx}$

$B = \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx} \right) \times \frac{1}{12e^{-6x}}$