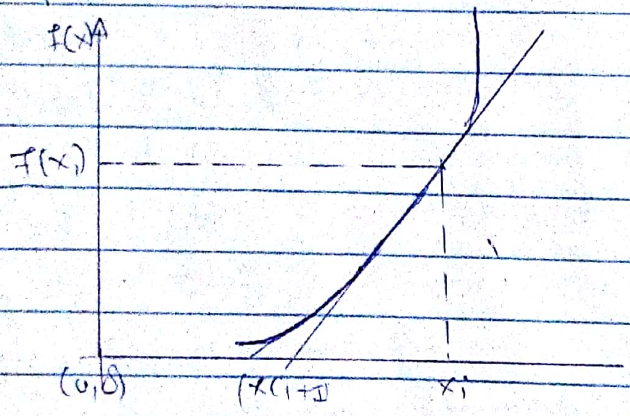
Victor Uchoni
16/ENG04/077
Electrical Electronics
ENG 382

If the maximum percentage absolute error desired is to be 1E−9, using Newton-Raphson's iteration method and initial guess value of 0.5, find the root of the function given in equation (1.1)

(i) Manually

(ii) with MATLAB

$f(x) = e^{-0.5x}(4-x) - 2$




$f'(x) = \dfrac{(f(x_i) - 0)}{x_i - (x_{i+1})}$

$f'(x_i)(x_i - (x_{(i+1)})) = f(x_{(i)})$

$f'(x_i) * (x_{(i)}) - f'(x)(x_{(i+1)}) = f(x_i)$

$x_{(i+1)} = \dfrac{f'(x_i) * (x_{(i)}) - f(x_i)}{f'(x_i)}$

$x_{(i+1)} = x_i - \dfrac{f(x_{(i)})}{f'(x_{(i)})}$

$f(x) = e^{-0.5x}(4-x) - 2$

$f(x) = 4e^{-0.5x} - (xe^{-0.5x}) - 2$

$f'(x) = -0.5 \times 4(e^{-0.5x}) - (x \times (-0.5e^{-0.5x}) + e^{-0.5x} \cdot 1) - 0$

$f'(x) = -0.5 \times 4e^{-0.5x} - (x\,0.5e^{-0.5x} + e^{-0.5x}) - 0$

$= -2e^{-0.5x} + x\,0.5e^{-0.5x} - e^{-0.5x}$

Therefore if $x_0 = 0.5$ as given $x_{(i+1)} = x_i - f(x_{(i)})$

when $i = 0$

$$x_{(0+1)} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_0 = 0.5$ $\quad x_1 = 0.5 - \left[\frac{e^{-0.5(0.5)}[4 - 0.5) - 2)}{(-2e^{-(0.5 \times 0.5)} + 0.5(0.5e^{-0.5 \times 0.5}) - e^{-0.5(0.5)}}\right]$

$$x_1 = 0.5 - \left[\frac{0.725803}{2.1417022}\right]$$

$$x_1 = 0.5 - (-0.338890) = 0.83889$$

$$\text{error} = e_s = \frac{x_{(i+1)} - x_{(i)}}{x_{(i+1)}}$$

when $i = 0$ $\quad e_s = \frac{x_1 - x_0}{x_1}$

$$e_s = \frac{0.83889 - 0.5}{0.83889} = 0.40397 \times 100 = 40.397\%$$

when $i = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.83889 - \left[\frac{e^{-(0.83889 \times 0.5)}[4 - (0.83889) - 2}{-2e^{-(0.5 \times 0.83889)} + (0.83889 \times 0.5 \times e^{(-0.5 \times 0.83889)}) - e^{(0.5 \times 0.83889)}}\right]$$

$$x_2 = 0.83889 - \left[\frac{0.478150}{-1.6640}\right] = 0.8555$$

when $i = 1$

$$\text{error} = \frac{x_2 - x_1}{x_2}$$

$$= \frac{0.8858 - 0.83887}{0.8858}$$

$$= 0.05295 \times 100$$

$$= 5.295\%$$

when $i = 2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.8858 - \left( \frac{e^{-0.8858 \times 0.5}(4 - 0.8858) - 2}{-2e^{-(-0.5 \times 0.8858)} + (0.8858 \times 0.5 \times e^{(-0.5 \times 0.8858)}) - (e^{-(0.5 \times 0.8858)})} \right)$$

$$x_3 = 0.8858 - \left[ \frac{-0.0001496}{-1.6426} \right]$$

$$0.8858 + 0.000091075$$

$$= 0.885891075$$

$$\text{error} = \frac{x_3 - x_2}{x_3}$$

$$= \frac{0.88589 - 0.8858}{0.88589}$$

$$= 0.0001075 = 1.015 \times 10^{-4}$$

$$= 0.01\%$$

when $i = 3$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = 0.88589 - \left( \frac{e^{-0.88589 \times 0.5} \times (4 \times 0.88589) - 2}{2e^{-(-0.5 \times 0.88589)} + (0.8859 \times 0.5 \times e^{(-0.5 \times 0.88589)}) - e^{-(0.5 \times 0.88589)}} \right)$$

$$x_4 = 0.88589$$

$$\text{error} = \frac{0.88589 - 0.88589}{0.88589}$$

$$= 0\%$$

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