

NELSON-UDU .A. ELAH

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PETROLEUM ENGINEERING

ENG 382

ASSIGNMENT 3

9/03/2019

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A

$$\begin{aligned}T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 &= 4 \\2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 &= 20 \\T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 &= -15 \\5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 &= -3 \\-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 &= 16 \\4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 &= -27\end{aligned}$$

Solution

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

Augmenting the matrix, we have

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

$$\text{for } a_{21} = 2 - (2)(1) = 0$$

$$a_{22} = -1 - (2)(1) = -3$$

$$a_{23} = 1 - (2)(-2) = 5$$

$$a_{24} = 2 - (2)(-1) = 0$$

$$a_{25} = 1 - (2)(2) = -3$$

$$a_{26} = -3 - (2)(3) = -9$$

$$a_{27} = 20 - (2)(4) = 12$$

Q₃₁

$$\text{for } Q_{31} : 1 - (1) \times 1 = 0$$

$$Q_{32} : 3 - (1) \times 1 = 2$$

$$Q_{33} : -3 - (1) \times (2) = -1$$

$$Q_{34} : -1 - (1) \times 1 = -2$$

$$Q_{35} : 2 - (1) \times (3) = -1$$

$$Q_{36} : 1 - (1) \times (-1) = 2$$

$$Q_{37} : -15 - (1) \times 4 = -19$$

$$\text{for } Q_{41} : 5 - (5) \times 1 = 0$$

$$Q_{42} : 2 - (5) \times 1 = -3$$

$$Q_{43} : -1 - (5) \times 2 = -9$$

$$Q_{44} : -1 - (5) \times 1 = -6$$

$$Q_{45} : 2 - (5) \times 3 = -13$$

$$Q_{46} : 1 - (5) \times -1 = 6$$

$$Q_{47} : -3 - (5) \times 4 = -23$$

$$\text{for } Q_{51} : -3 - (-3) \times 1 = 0$$

$$Q_{52} : -1 - (-3) \times 1 = 2$$

$$Q_{53} : 2 - (-3) \times -2 = -4$$

$$Q_{54} : 3 - (-3) \times 1 = 6$$

$$Q_{55} : 1 - (-3) \times 3 = 10$$

$$Q_{56} : 3 - (-3) \times 1 = 0$$

$$Q_{57} : 16 - (-3) \times 4 = 28$$

$$\text{for } Q_{61} : 4 - 4 \times 1 = 0$$

$$Q_{62} : -3 - (4) \times 1 = -1$$

$$Q_{63} : -1 - (4) \times -2 = 7$$

$$Q_{64} : -6 - (4) \times 1 = -10$$

$$Q_{65} : -3 - (4) \times 3 = -15$$

$$Q_{66} : -2 - (4) \times -1 = 2$$

$$Q_{67} : -27 - (4) \times 4 = -43$$

Q14

forward elimination

$$\left[\begin{array}{ccccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & 1 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & 1 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 1 & 28 \\ 0 & -1 & 9 & -10 & -15 & 2 & 1 & -43 \end{array} \right]$$

for a_{32} in Row 3 $\frac{a_{32}}{a_{22}} = -\frac{2}{3}$

$$\therefore a_{32} = 2 - \left(-\frac{2}{3}\right) \times -3 = 0$$

$$a_{33} = -1 - \left(-\frac{2}{3}\right) \times 5 = \frac{7}{3}$$

$$a_{34} = -2 - \left(-\frac{2}{3}\right) \times 0 = -2$$

$$a_{35} = -1 - \left(-\frac{2}{3}\right) \times -5 = -\frac{13}{3}$$

$$a_{36} = 2 - \left(-\frac{2}{3}\right) \times -1 = \frac{4}{3}$$

$$a_{37} = -15 - \left(-\frac{2}{3}\right) \times 12 = -11$$

for a_{42} in Row 4 $\frac{a_{42}}{a_{22}} = -\frac{3}{-2} = 1$

$$\therefore a_{42} = -3 - (-1) \times -3 = 0$$

$$a_{43} = 9 - (-1) \times 5 = 14$$

$$a_{44} = -6 - (-1) \times 0 = -6$$

$$a_{45} = -13 - (-1) \times -5 = -18$$

$$a_{46} = 6 - (-1) \times -1 = 7$$

$$a_{47} = -23 - (-1) \times 12 = -35$$

for a_{52} in Row 5 $\frac{a_{52}}{a_{22}} = -\frac{1}{3} = \frac{1}{3}$

$$\therefore a_{52} = -1 - \left(\frac{1}{3}\right) \times -3 = 0$$

$$a_{53} = 9 - \left(\frac{1}{3}\right) \times 5 = \frac{22}{3}$$

$$a_{54} = -10 - \left(\frac{1}{3}\right) \times 0 = -10$$

$$a_{55} = -15 - \left(\frac{1}{3}\right) \times -5 = -\frac{40}{3}$$

$$a_{56} = 2 - \left(\frac{1}{3}\right) \times -1 = \frac{7}{3}$$

$$a_{57} = 43 - \left(\frac{1}{3}\right) \times 12 = 47$$

Stage 2

$$\left[\begin{array}{ccccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} & 1 & -11 \\ 0 & 0 & 9 & -6 & -8 & 7 & 1 & -35 \\ 0 & 0 & \frac{22}{3} & 6 & \frac{20}{3} & -\frac{2}{3} & 1 & -36 \\ 0 & 0 & \frac{27}{3} & -10 & -\frac{40}{3} & \frac{7}{3} & 1 & -47 \end{array} \right]$$

Q44

for $Q_{43} = 0$ Row 3 $Q_{43}/Q_{33} = 4 \div 7 = \frac{4}{7}$

for $Q_{44} = 4 - (\frac{12}{7}) \times \frac{4}{3} = 0$

$Q_{45} = -6 - (\frac{12}{7}) \times 2 = -\frac{18}{7}$

$Q_{46} = -8 - (\frac{12}{7}) \times \frac{8}{3} = -\frac{24}{7}$

$Q_{47} = 7 - (\frac{12}{7}) \times \frac{4}{3} = \frac{33}{7}$

$Q_{48} = -35 - (\frac{12}{7}) \times 11 = -\frac{113}{7}$

for $Q_{53} = 0$ Row 5 $Q_{53}/Q_{33} = -\frac{2}{3} \div \frac{4}{3} = -\frac{2}{4}$

$Q_{54} = -\frac{2}{3} - (-\frac{2}{4}) \times \frac{4}{3} = 0$

$Q_{55} = 6 - (-\frac{2}{4}) \times 2 = \frac{38}{4}$

$Q_{56} = \frac{20}{3} - (-\frac{2}{4}) \times \frac{12}{3} = \frac{38}{4}$

$Q_{57} = -\frac{2}{3} - (-\frac{2}{4}) \times \frac{4}{3} = -\frac{2}{3}$

$Q_{58} = 36 - (-\frac{2}{4}) \times 11 = \frac{230}{4}$

for $Q_{63} = 0$ $Q_{63}/Q_{33} = \frac{22}{3} \div \frac{4}{3} = \frac{22}{4}$

$Q_{64} = \frac{22}{3} - (\frac{22}{4}) \times \frac{4}{3} = 0$

$Q_{65} = 10 - (\frac{22}{4}) \times 2 = -\frac{26}{4}$

$Q_{66} = -\frac{40}{3} - (\frac{22}{4}) \times \frac{12}{3} = \frac{2}{4}$

$Q_{67} = \frac{7}{3} - (\frac{22}{4}) \times \frac{4}{3} = -\frac{13}{4}$

$Q_{68} = -47 - (\frac{22}{4}) \times 11 = -\frac{87}{4}$

Step 3

1	1	-2	1	3	-1	1	4
0	-3	5	0	-5	-1	1	-12
0	0	$\frac{7}{3}$	-2	$-\frac{12}{3}$	$\frac{4}{3}$	1	-11
0	0	0	$-\frac{18}{2}$	$\frac{4}{2}$	$\frac{38}{4}$	1	$-\frac{113}{4}$
0	0	0	$\frac{35}{4}$	$\frac{28}{4}$	$\frac{38}{4} - \frac{2}{4}$	1	$\frac{230}{4}$
0	0	0	$-\frac{26}{4}$	$\frac{2}{4}$	$-\frac{13}{4}$	1	$-\frac{87}{4}$

for Q_{54} i.e. Row 4 $Q_{54}/Q_{44} = \frac{38}{4} \div -\frac{18}{4} = -\frac{19}{9}$

$Q_{55} = \frac{38}{4} - (-\frac{19}{9}) \times -\frac{18}{4} = 0$

$Q_{56} = \frac{38}{4} - (-\frac{19}{9}) \times \frac{38}{4} = \frac{38}{9}$

$Q_{57} = -\frac{2}{4} - (-\frac{19}{9}) \times \frac{33}{4} = \frac{25}{3}$

$Q_{58} = \frac{230}{4} - (-\frac{19}{9}) \times -\frac{113}{4} = -\frac{11}{9}$

Opt

$$\begin{aligned} \text{for } q_{64} &= 0 \quad q_{64}/q_{44} = -26/7 \div -18/2 = 13/7 \\ \text{for } q_{64} &= -26/7 - (13/7) \times 18/7 = 0 \\ q_{65} &= 2/7 - (13/7) \times 4/7 = 10/9 \\ q_{66} &= -13/7 - (13/7) \times 33/7 = -26/3 \\ q_{67} &= -87/7 - (13/7) \times 113/7 = 98/7 \end{aligned}$$

stage 4

1	1	-2	1	-8	3	-1	1	4
0	-3	5	0	-18/7	-5	-1	1	12
0	0	7/3	-2	-4/7	-13/3	4/3	1	-11
0	0	0	-18/7	-89/7	-4/7	33/7	1	-113/7
0	0	0	0	38/7	27/3			-11/5
0	0	0	0	10/7	-26/3			98/7

$$\begin{aligned} \text{for } q_{65} &= 0 \quad q_{65}/q_{55} = 10/7 \times 7/38 = 10/38 \\ \therefore q_{65} &= 10/7 - (10/38) \times 38/7 = 0 \\ q_{66} &= -26/3 - (10/38) \times 27/3 = -213/15 \\ q_{67} &= 98/7 - (10/38) \times 113/7 = 213/15 \end{aligned}$$

final stage

1	1	-2	1	3	-1	1	4
0	-3	5	0	-5	-1	1	12
0	0	7/3	-2	13/3	4/3	1	-11
0	0	0	-18/7	-4/7	23/7		-113/7
0	0	0	0	38/7	27/3		-11/5
0	0	0	0	0	-213/15		213/15

using Back substitution

$$\begin{aligned} T_6 &= 0T_1 + 0T_2 + 0T_3 + 0T_4 + 0T_5 + \frac{-213}{15}T_6 = \frac{213}{15} \\ \frac{-213}{15}T_6 &= \frac{213}{15} \\ \therefore T_6 &= \frac{213}{15} \div \frac{-213}{15} = \frac{213}{15} \times \frac{15}{-213} \\ T_6 &= -1 \end{aligned}$$

$$\begin{aligned} T_5 &= 0T_1 + 0T_2 + 0T_3 + 0T_4 + \frac{38}{7}T_5 + \frac{27}{3}T_6 = -11/5 \\ \frac{38}{7}T_5 + \frac{27}{3}(-1) &= -11/5 \\ \therefore T_5 &= \frac{-11/5 + 27/3}{38/7} \\ &= \frac{96}{5} \times \frac{7}{38} \end{aligned}$$

$$T_5 = 2$$

Q1

find the elimination

1	1	-2	1	3	-1	1
0	-3	5	0	-5	-1	12
0	2	-1	-2	-1	2	-19
0	-3	9	-6	-13	6	-23
0	2	-4	-8	10	0	26
0	-1	9	-10	-15	2	-43

$$\text{for } a_{32} = \frac{a_{32}}{a_{22}} = -\frac{2}{3}$$

$$\therefore a_{32} = 2 - \left(-\frac{2}{3}\right) \times -3 = 0$$

$$a_{33} = -1 - \left(-\frac{2}{3}\right) \times 5 = \frac{7}{3}$$

$$a_{34} = -2 - \left(-\frac{2}{3}\right) \times 0 = -2$$

$$a_{35} = -1 - \left(-\frac{2}{3}\right) \times -1 = \frac{4}{3}$$

$$a_{36} = -17 - \left(-\frac{2}{3}\right) \times 12 = -4$$

$$T_4 = 0T_1 + 0T_2 + 0T_3 + \left(-\frac{18}{1}\right)T_4 - \frac{4}{3}T_5 + \frac{33}{1}T_6 = -\frac{113}{3}$$

$$-\frac{18}{1}T_4 - \frac{4}{3}T_5 + \frac{33}{1}T_6 = -\frac{113}{3}$$

$$\therefore T_4 = \left(-\frac{113}{3} + \frac{4}{3}T_5 - \frac{33}{1}T_6\right) \div -\frac{18}{1}$$

$$T_4 = -\frac{113}{3} \times \frac{1}{-18} = 4$$

$$\therefore T_4 = 4$$

$$T_3 = 0T_1 + 0T_2 + \frac{9}{3}T_3 - 2T_4 - \frac{13}{3}T_5 + \frac{4}{3}T_6 = -11$$

$$= \frac{9}{3}T_3 - 2(4) - \frac{13}{3}(2) + \frac{4}{3}(-1) = -11$$

$$\therefore T_3 = \left(-11 + 8 + \frac{26}{3} + \frac{4}{3}\right) \div \frac{9}{3}$$

$$T_3 = \frac{7}{1} \times \frac{3}{2} = 3$$

$$\therefore T_3 = 3$$

$$T_2 = 0T_1 + -3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 = 12$$

$$= -3T_2 + 5T_3 - 5T_5 - (1) = 12$$

$$T_2 = (12 - 5(3) + 5(2) - 1) \div -3$$

$$T_2 = 6 \div -3 = -2$$

$$\therefore T_2 = -2$$

$$T_1 = T_4 + T_2 - 2T_3 + T_4 + 3T_5 + T_6 = 4$$

$$= T_1 = 4 + 2 + 6 - 4 - 6 = 1$$

$$T_1 = 1$$

Therefore the value of the temperatures are given

$$T_1 = 1^{\circ}\text{C}$$

$$T_2 = 2^{\circ}\text{C}$$

$$T_3 = 3^{\circ}\text{C}$$

$$T_4 = 4^{\circ}\text{C}$$

$$T_5 = -2^{\circ}\text{C}$$

$$T_6 = -1^{\circ}\text{C}$$