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PETROLEUM ENGINEERING

ENG 382

ASSIGNMENT 2

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ASSIGNMENT 2

A) manually

$f(x) = e^{-0.5x}(4-x) - 2$

initial guess value $= 0.5 = x_0$

max percentage error $= 1E-9$

solution

$f(x) = (4-x)e^{-0.5x} - 2$

To find the root

when $x = 0$, $f(x) = (4-0)$

To find the root

when $x = 0$, $f(x) = (4-0)e^{-0.5(0)} - 2 = 2$

$f(x) = 2$

when $x = 1$, $f(x) = (4-1)e^{-0.5(1)} - 2 = -0.180408$

$f(x) \approx -0.180$

$f(x) = e^{-0.5x}(4-x) - 2$

$= 4e^{-0.5x} - xe^{-0.5x} + 2$

using product rule

$f'(x) = \frac{d}{dx}\left[e^{-0.5x}(4-x)\right] - \frac{d(2)}{dx}$

$= e^{-0.5x}\frac{d}{dx}(4-x) + (4-x)\cdot\frac{d}{dx}(e^{-0.5x}) - 0$

$= e^{-0.5x} \cdot 1 + (4-x) \cdot -0.5e^{-0.5x}$

$= 4 \cdot -0.5e^{-0.5x} + x - 0.5e^{-0.5x} - e^{-0.5x}$

$= x \cdot 0.5e^{-0.5}x - 2e^{-0.5x} - e^{-0.5x}$

$f'(x) = 0.5xe^{-0.5x} - 3e^{-0.5x}$

$= e^{-0.5x}[0.5x - 3$

using Newton raphson

$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

for % err. $\left[\frac{x_{k+1} - x_k}{x_{k+1}}\right] \times 100\%$

for iter 1,

let $x_k = 0.5 = x_0$

$f(x_0) = (4 - 0.5)e^{-0.5(0.5)} - 2$

$f(x_0) = 0.225802407$

Q4

$f'(x_0) = e^{-0.5}(0.5)[(0.5\times0.5)-3]$

$= -2.141702153$

$x_{k+1} = 0.5 - \frac{0.725802744}{-2.141702153}$

$x_{k+1} = 0.8388906$

% abs error $= \left[\frac{0.8388906 - 0.5}{0.8388906}\right]\times 100$

$= 40.35742558 \simeq 40.4\%$

for Iter 2

$x_k = 0.83889606 = x_1$

$f(x_1) = (4-0.83889606)e^{-0.5(0.83889606)} - 2$

$= 0.078149299794$

$f'(x_1) = e^{-0.5\times(0.83889606)}[0.5\times0.83889606)-3]$

$= -1.696486032$

$x_{k+1} = 0.83889606 - \left[\frac{0.078149299794}{-1.696486032}\right]$

$x_{k+1} = 0.884956003$

% abs error $= \left[\frac{0.884956003 - 0.83889606}{0.884956003}\right]\times 100$

$= 5.205388094\%$

for Iter 3

$x_k = 0.884956003 = x_2$

$f(x_2) = (4-0.884956003)e^{-0.5(0.884956003)} - 2$

$= 0.012365751$

$f'(x_2) = e^{-0.5\times(0.884956003)}((0.5\times0.884956003)-3]$

$= -1.643060762$

$x_{k+1} = 0.884956003 - \left[\frac{0.012365751}{-1.643060762}\right]$

$x_{k+1} = 0.8857086054$

% abs error $= \left[\frac{0.8857086054 - 0.884956003}{0.8857086054}\right]\times 100$

$= 0.084912039\%$

Iter 4

$x_k = 0.885708605 = x_3$

$f(x_3) = (4 - 0.885708605)e^{-0.5(0.885708605)} - 2$

$= 3.23521411 \times 10^{-7}$

$f'(x_3) = e^{-0.5 \times (0.885708605)}[(0.5 \times 0.885708605) - 3]$

$= -1.642200925$

$x_{k+1} = 0.885708605 - \left[\frac{3.23521411 \times 10^{-7}}{-1.642200925}\right]$

$= 0.885708802$

$\%$ abs error $= \left[\frac{0.885708802 - 0.885708605}{0.885708802}\right] \times 100$

$= 2.224261137 \times 10^{-5}$

Iter 5

$x_k = 0.885708802 = x_4$

$f(x_4) = (4 - 0.885708802)e^{-0.5}(0.885708802) - 2$

$= 7.851 \times 10^{-12}$

$f'(x_4) = e^{-0.5 \times (0.885708802)} e^{-0.5}(0.885708802) - 2$

$= -1.642200704$

$x_{k+1} = 0.885708802 - \frac{7.851 \times 10^{-12}}{-1.642200929}$

$= 0.885708802$

$\%$ abs error $= \left[\frac{0.885708802 - 0.885708802}{0.885708802}\right] \times 100$

$= 0\%$

Summary of Results Obtained

i	x_k	$f(x_k)$	$f'(x_k)$	x_{k+1}	$\%$ abs error
1	0.5	0.7258027407	-2.141702153	0.838870606	40.35747275
2	0.838870606	0.07514952754	-1.696486032	0.884955003	5.205385054
3	0.884955003	0.001236857519	-1.643060762	0.885708605	0.08497203712
4	0.885708605	$3.23521411 \times 10^{-7}$	-1.642200925	0.885708802	$2.224261137 \times 10^{-5}$
5	0.885708802	7.851×10^{-12}	-1.642200704	0.885708802	0