

① Using an initial guess vector of $m_0 = [0, 0, 0]$ determine the values of the variables in equation

② Using Jacobi iterative method manually showing only three iterations

Soln

$$\begin{bmatrix} 6m_1 & -2m_2 & +m_3 & = & 9 \\ -2m_1 & +10m_2 & -2m_3 & = & 12 \\ -2m_1 & -5m_2 & +10m_3 & = & 18 \end{bmatrix}$$

$$m_1 = 0.2m_2 - 0.1m_3 + 0.9$$

$$m_2 = 0.2m_1 + 0.2m_3 + 1.2$$

$$m_3 = 0.2m_1 + 2.5m_2 + 1.8$$

$$\begin{bmatrix} m_1^{(k)} \\ m_2^{(k)} \\ m_3^{(k)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} m_1^{(k-1)} \\ m_2^{(k-1)} \\ m_3^{(k-1)} \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

$$m^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

First Iteration

$$\begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

$$[(0 \times 0.2) + (0.2 \times 0) + (-0.1 \times 0)] + 0.9 = 0.9$$

$$[(0.2 \times 0) + (0 \times 0) + (0.2 \times 0)] + 1.2 = 1.2$$

$$[(0.5 \times 0) + (0.5 \times 0) + (0 \times 0)] + 1.8 = 1.8$$

Second Iteration

$$\begin{bmatrix} M_{1(2)} \\ M_{2(2)} \\ M_{3(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.96 \\ 1.74 \\ 2.58 \end{bmatrix}$$

$$[(0 \times 0.9) + (0.2 \times 1.2) + (-0.1 \times 1.8)] + 0.9 = 0.96$$

$$[(0.2 \times 0.9) + (0 \times 1.2) + (0.2 \times 1.8)] + 1.2 = 1.74$$

$$[(0.2 \times 0.9) + (0.5 \times 1.2) + (0 \times 1.8)] + 1.8 = 2.58$$

Third Iteration

$$\begin{bmatrix} M_{1(3)} \\ M_{2(3)} \\ M_{3(3)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.96 \\ 1.74 \\ 2.58 \end{bmatrix} + \begin{bmatrix} 0.96 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.996 \\ 1.908 \\ 2.862 \end{bmatrix}$$

$$[(0 \times 0.96) + (0.2 \times 1.74) + (-0.1 \times 2.58)] + 0.9 = 0.996$$

$$[(0.2 \times 0.96) + (0 \times 1.74) + (0.2 \times 2.58)] + 1.2 = 1.908$$

$$[(0.2 \times 0.96) + (0.5 \times 1.74) + (0 \times 2.58)] + 1.8 = 2.862$$