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CHEMICAL ENGINEERING
17/ENG01/008

Assignment 1

(a) Define a differential equation and give two examples

ANSWER: A differential Equation is a relationship between an independent variable, x a dependent variable, Y and one or more derivatives of Y with respect to x

Example: (1) $2xy\frac{d^2y}{dx^2} + y\frac{dy}{dx} + 3e^{3x} = 0$

(2) $4y\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 18x^2 = \cos 4y$

(b) An expression has been obtained for an engineering system to be given in: $Y = Ae^{-4x} + Be^{-6x}$

(i) What is the order of the differential equation that can be formed from the expression.

Answer: Second Order differential Equation

(ii) Give a reason for your answer

ANSWER: This is because the expression contains two arbitrary constants.

(iii) Form a differential equation from the expression

$$Y = Ae^{-4x} + Be^{-6x} \quad \text{—(1)}$$

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{—(2)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{—(3)}$$

From equation (2)

$$\frac{dy}{dx} + 4Ae^{-4x} + 6Be^{-6x} = 0$$

$$4Ae^{-4x} = -\frac{dy}{dx} - 6Be^{-6x}$$

$$A = \frac{1}{4}e^{-4x}\left[-\frac{dy}{dx} - 6Be^{-6x}\right] \quad \text{—(4)}$$

Input equation (4) into (3.)

$$\frac{d^2y}{dx^2} = \frac{4}{6e^{-6x}}\left[\frac{1}{4e^{-4x}}\left[\frac{-dy}{dx} - 6Be^{-6x}\right]\right] + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 4\left[\frac{-dy}{dx} - 6Be^{-6x}\right] + 36Be^{-6x}$$

$$d^2y/dx^2 = -4dy/dx - 24Be^{-6x} + 36Be^{-6x} = -4dy/dx + 12Be^{-6x}$$

$$B = \left[d^2y/dx^2 + 4dy/dx\right]^{1}/12e^{-6x} \quad \text{---- (5.)}$$

Recall from equation 2:

$$dy/dx = -4Ae^{-4x} - 6Ae^{-6x}$$

$$dy/dx + 4Ae^{-4x} + 6Be^{-6x} = 0$$

$$6Be^{-6x} = \left[-dy/dx - 4Ae^{-4x}\right]$$

$$B = \frac{1}{6e^{-6x}}\left[\frac{-dy}{dx} - 4Ae^{-4x}\right] \quad \text{---- (6.)}$$

Input equation (6.) to (3.)

$$d^2y/dx^2 = 16Ae^{-4x} + \frac{6}{36e^{-6x}} \times \left[\frac{1}{6e^{-6x}}\left[\frac{-dy}{dx} - 4Ae^{-4x}\right]\right]$$

$$d^2y/dx^2 = 16Ae^{-4x} - 6dy/dx - 24Ae^{-4x} = -6dy/dx - 8Ae^{-4x}$$

$$d^2y/dx^2 + 6dy/dx + 8Ae^{-4x} = 0, \quad 8Ae^{-4x} = -d^2y/dx^2 - 6dy/dx$$

$$A = \frac{1}{8e^{-4x}}\left[\frac{-d^2y}{dx^2} - \frac{6dy}{dx}\right] \quad \text{---- (7.)}$$

Substitute equation (7.) and (5.) into Equation (1.)

$$Y = \frac{e^{-4x}}{8e^{-4x}}\left[\frac{-d^2y}{dx^2} - \frac{6dy}{dx}\right] + \left[\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right]\frac{e^{-6x}}{12e^{-6x}}$$

$$Y = -d^2y/8dx^2 - 3/4\,dy/dx + d^2y/12dx^2 + dy/3dx$$

$$Y = -\frac{d^2y}{24dx^2} - \frac{5dy}{12dx} \quad \text{---- (8.)}$$

Multiplying through by 24

$$24Y = -\frac{d^2y}{dx^2} - 10\frac{dy}{dx}$$

$$24y + d^2y/dx^2 + 10dy/dx = 0$$

$$\left[\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0\right]$$