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Electrical Engineering
Engineering Maths Assignment 1

a) A differential equation is an equation which shows the relationship between the independent variable, dependend variable and one or more derivatives of the dependent variable

Example

i) $\frac{dy}{dx} - 5y = x + 1$ ii) $(x+2)\frac{dy}{dx} = y^2$

b) $y = Ae^{-4x} + Be^{-6x}$ - 2nd order differential

i) Order of the differential equation

ii) It is Second order differential equation because it has 2 arbitary constants (A and B)

iii) $y = Ae^{-4x} + Be^{-6x}$

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$

Let $R = Ae^{-4x}$, $S = Be^{-6x}$

$y = R + S$

$\therefore \frac{dy}{dx} = -4R - 6S$ $\frac{d^2y}{dx^2} = 16R + 36S$

$\frac{dy}{dx} = -4R - 6S$ — 1) $\frac{d^2y}{dx^2} = 16R + 36S$ — 2)

multiply 1) by 36

multiply 2) by 6

$36\frac{dy}{dx} = -144R - 216$ — 3)

$-6\frac{d^2y}{dx^2} = 96R - 216S$ — 4)

Subtract 4) from 3

$36\frac{dy}{dx} - 6\frac{d^2y}{dx^2} = -144R - 216S$ — 3)

$36\frac{dy}{dx} = -144R - 216S$ — 5)

$\therefore 6\frac{d^2y}{dx^2} = -96R - 216S$ — 4)

multiply 4) by -1

Subtract 4) from 3

$\Rightarrow 36\frac{dy}{dx} + 6\frac{d^2y}{dx^2} = -48R$

$\Rightarrow R = -\frac{3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2}$

put R into eqn 1)

$\Rightarrow \frac{dy}{dx} = -4\left(-\frac{3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2}\right)$ — 6)

$\Rightarrow S = \frac{1}{3}\frac{dy}{dx} + \frac{1}{12}\frac{d^2y}{dx^2}$

$\because y = R + S$

$\Rightarrow y = \left(-\frac{3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2}\right) + \left(\frac{1}{3}\frac{dy}{dx} + \frac{1}{12}\frac{d^2y}{dx^2}\right)$

$\Rightarrow y = \frac{-5}{12}\left(\frac{dy}{dx}\right) - \frac{1}{24}\frac{d^2y}{dx^2}$

$\Rightarrow y = \frac{1}{12}\left(-5\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right)$

$\Rightarrow 12y = -5dy/dx - 1/2\, d^2y/dx^2$

$\Rightarrow 24y = -10\, dy/dx - d^2y/dx^2$

$\Rightarrow \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$