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Mechanical engineering

1a A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivative of y with respect to x

Examples: (i) $\frac{dy}{dx} = 2 + \frac{y}{x}$

(ii) $\frac{dy}{dx} = y + \frac{y}{x}$

b $y = Ae^{-4x} + Be^{-6x}$

i A second order differential equation

ii A second Order differential equation can be formed because it contains 2 constants in the degenerate equation

iii $y = Ae^{-4x} + Be^{-6x}$

solution

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$ —(i)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ —(ii)

solving equ(i) and (ii) simultaneously

Multiply equ(i) by 6

$\therefore 6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$ —(iii)

$\frac{d^2y}{dx^2} = +16Ae^{-4x} - 36Be^{-6x}$ —(iv)

$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$

$\therefore A = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}}$ —(v)

substituting equ(v) into equ(i)

$\frac{dy}{dx} = 4\left(\frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$

$$\frac{dy}{dx} = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

multiply through by 2

$$\therefore 2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2\frac{dy}{dx} - 6\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B$$

$$\therefore \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} = B$$

substitute A and B into the degenerate equation

$$\therefore y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-6x}} \times e^{-6x} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12}$$

$$y = \frac{-72\frac{dy}{dx} - 12\frac{d^2y}{dx^2} + 32\frac{dy}{dx} + 8\frac{d^2y}{dx^2}}{96}$$

$$y = \frac{-40\frac{dy}{dx} - 4\frac{d^2y}{dx^2}}{96}$$

$$96y = -40\frac{dy}{dx} - 4\frac{d^2y}{dx^2}$$

$$24y = -10\frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\therefore \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$