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mechanical engineering
200 level 18/engo6/074

1 Given that $\hat{f} = x^2 i + (3x+2)j + \sin x k$;
find (a) $\frac{df}{dx}$ (b) $\frac{d^2f}{dx^2}$ (c) $\left|\frac{df}{dx}\right|$ (d) $\frac{d}{dx}(f \cdot f)$ at $x=1$

Solution

$f = x^2 i + (3x+2)j + \sin x k$

a) $\frac{df}{dx} = 2x i + (3) j + \cos x k$

$\therefore \frac{df}{dx} = 2x i + 3j + \cos x k$

b) $\frac{d^2f}{dx^2} = 2i - \sin x k$

c) $\left|\frac{df}{dx}\right| = \sqrt{(2x)^2 + (3)^2 + (\cos x)^2}$

$= \sqrt{4x^2 + 9 + \cos^2 x}$

At $x = 1 \Rightarrow \sqrt{4(1)^2 + 9 + \cos^2(1)}$

$= \sqrt{4 + 9 + 0.999}$

$= \sqrt{13.999} = 3.74$

d $\frac{d}{dx}(f \cdot f) \rightarrow (f \cdot f) = [(x^2 i) + (3+2)j + \sin x k] \cdot [(x^2) i + (3x+2)j + \sin x k]$

$= x^4 + (9x^2 + 6x + 6x + 4) + \sin^2 x$

$= x^4 + 9x^2 + 12x + 4 + \sin^2 x$

$\therefore \frac{d}{dx}(f \cdot f) = 4x^3 + 18x + 12 + 2\sin x \cos x$

$\frac{d}{dx}(f \cdot f)_{at\, x=1} = 4(1)^3 + 18(1) + 12 + 2\sin(1)\cos(1)$

$= 4 + 18 + 12 + 2(0.0175)(0.999)$

$= 4 + 18 + 12 + 0.035$

$= 34.035$

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2) if $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$. Determine (a) $\frac{dr}{dt}$ (b) $\frac{d^2r}{dt^2}$ (c) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

Solution

a) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t}k$

b) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t}k$

c) $\left|\frac{d^2r}{dt^2}\right|$ at $=0$

$\therefore r = 2i + [18\sin(3)(0)]j + 12e^{2(0)}k$

$= 2i + 18\sin 0\, j + 12e^{0}k$

$= 2i + 18(0)j + 12(1)k$

$= 2i + 12k$

$\therefore \left|\frac{d^2r}{dt^2}\right| = \sqrt{(2)^2 + (12)^2}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{4+144} = \sqrt{148} = 12.17$