Maths Assignment ①

$y = Ae^{-4x} + Be^{-6x}$.

ⓐ 2nd Order Differential Equation.

Why because it has Presence of two Constant Variables A and B.

ⓑ To form a differential eqn.

① → ① $y = Ae^{-4x} + Be^{-6x}$ { Recall that when there are two Constant Variables we are to differentiate twice

This is that is.

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$ -- ②

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ --- ③.

To get Constant A

Multiply eqn ② by x6

Then we have.

$6\frac{d^2y}{dx^2} = 6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$ ---- ④.

We Multiplied by x6 because of the number with Respect to Constant B.

Bringing eqn ④ and ⑤ and solving them Simultaenously.

~~Bringing eqn 4 and 5~~

$$\frac{6dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

Using elimination Method ~~(eqn)~~ (Addition):

$$\frac{6dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

Making A Subject of formula.

$$\frac{-1}{8e^{-4x}}\left[\frac{6dy}{dx} + \frac{d^2y}{dx^2}\right] = A$$

For Constant B.

Multiply eqn ① by 4 { Because it is the Number attached to Constant A. }

Then we have.

$$4\frac{dy}{dx} = -16Ae^{-4x} - 24Be^{-6x} \quad \text{--- (5)}$$

Bringing eqn (5) and (3) together and Solving them Simultaneously.

$$4\frac{dy}{dx} = -16Ae^{-4x} - 24Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$4\frac{dy}{dx} + \frac{d^2y}{dx^2} = 12Be^{-6x}.$$

Making B Subject of formula.

$$\frac{1}{12e^{-6x}}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right] = B.$$

Recall the eqn

$$y = Ae^{-4x} + Be^{-6x}$$

$$y = \frac{-1}{8e^{-4x}}\left[6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]e^{-4x} + \frac{1}{12e^{-6x}}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]e^{-6x}$$

$$y = \frac{-1}{8}\left[6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right] + \frac{1}{12}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]$$

Opening and Simplifying the bracket.

$$y = -\frac{6}{8}\frac{dy}{dx} - \frac{d^2y}{8dx^2} + \frac{4}{12}\frac{dy}{dx} + \frac{1}{12}\frac{d^2y}{dx^2}.$$

$$y = -\frac{3}{4}\cdot\frac{dy}{dx} - \frac{1}{8}\cdot\frac{d^2y}{dx^2} + \frac{1}{3}\cdot\frac{dy}{dx} + \frac{1}{12}\cdot\frac{d^2y}{dx^2}.$$

Collecting like terms

$$y = -\frac{3}{4}\cdot\frac{dy}{dx} + \frac{1}{3}\cdot\frac{dy}{dx} - \frac{1}{8}\cdot\frac{d^2y}{dx^2} + \frac{1}{12}\cdot\frac{d^2y}{dx^2}.$$

$$y = \frac{-9+4}{12}\cdot\frac{dy}{dx} - \frac{-3+2}{24}\cdot\frac{d^2y}{dx^2}.$$

$$y = -\frac{5}{12}\cdot\frac{dy}{dx} + \frac{1}{24}\cdot\frac{d^2y}{dx^2}$$

Multiplying all through by $2x$

$$2xy = -10x\frac{dy}{dx} + x^2\frac{d^2y}{dx^2} \quad \text{--- Answer}$$