

Ass 1

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P/EN 100/001
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Q1) A differential equation is a relationship between an independent variable, a dependent variable, y , and one or more derivatives of y with respect to x .

$$\text{Example - } 2y \frac{dy}{dx} - x^2 \frac{dy}{dx} + 5x = 0$$

$$\text{Q2) } x \frac{dy}{dx} + y \frac{dy}{dx} + e^{2x} = \sin x$$

$$\text{Q3) } y = Ae^{-4x} + Be^{-6x} \dots \text{Q}$$

Q4) Eqn Q is a second order differential equation

~~Q5) This is a second order differential equation~~

Q6) There is no term here are two arbitrary constants

$$\text{Q7) } y = Ae^{-4x} + Be^{-6x} \dots \text{Q}$$

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \dots \text{Q}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \dots \text{Q}$$

$$\text{From eqn Q, } 4Ae^{-4x} = -\frac{dy}{dx} - 6Be^{-6x}$$

$$\therefore A = \left(-\frac{dy}{dx} - 6Be^{-6x} \right) \frac{1}{4e^{-4x}} \dots \text{Q}$$

Sub Q into Q

$$\frac{d^2y}{dx^2} = 16 \left(-\frac{dy}{dx} - 6Be^{-6x} \right) e^{-4x} + 36Be^{-6x}$$

$$\frac{d^2 y}{dx^2} = -\frac{4y}{x^2} - 24xe^{-1/x} + 54Be^{-2/x}$$

$$\frac{d^2 y}{dx^2} + \frac{4y}{x^2} = 0 \quad \text{--- (1)}$$

$$\therefore B = \left(\frac{d^2 y}{dx^2} + \frac{4y}{x^2} \right) \cdot \frac{1}{12e^{-2/x}} \quad \text{--- (2)}$$

Sub eqn (1) into eqn (2)

$$A = \left(-\frac{dy}{dx} - 6 \left(\frac{dy}{dx} + \frac{y}{x} \right) \cdot \frac{1}{12e^{-1/x}} \right) \cdot \frac{1}{4e^{-1/x}}$$

$$= \left(-\frac{dy}{dx} - \frac{1}{2} \left(\frac{dy}{dx} + \frac{y}{x} \right) \right) \cdot \frac{1}{4e^{-1/x}}$$

$$= \left(-\frac{3}{4} \frac{dy}{dx} - \frac{y}{2x} \right) \cdot \frac{1}{4e^{-1/x}} \quad \text{--- (3)}$$

Sub eqn (2) into (3)

$$y = 4e^{-1/x} + 3e^{-2/x} \quad \text{--- (4)}$$

$$y = \left(-\frac{3}{4} \frac{dy}{dx} - \frac{y}{2x} \right) \cdot \frac{1}{4e^{-1/x}} \cdot e^{-1/x} + \left(\frac{dy}{dx} + \frac{y}{x} \right) \cdot \frac{1}{12e^{-2/x}} \cdot e^{-2/x}$$

$$= \frac{1}{4} \left(-\frac{3}{4} \frac{dy}{dx} - \frac{y}{2x} \right) + \frac{1}{12} \left(\frac{dy}{dx} + \frac{y}{x} \right)$$

$$= \frac{3}{16} \frac{dy}{dx} - \frac{y}{8x} + \frac{dy}{dx} + \frac{y}{12x}$$

$$y = \frac{y}{24x} - \frac{y}{8x}$$

$$y = \left(-\frac{5}{24} \frac{dy}{dx} - \frac{y}{24x} \right) \cdot \frac{1}{12}$$