

Assignment 1

- (a) Define differential eqn and give two examples  
 (b) An expression has been obtained for an engineering system to be as given  

$$y = Ae^{-4x} + Be^{-6x}$$
- What is the order of the differential eqn that can be formed from the expression
  - Give a reason for your answer in (i)
  - Form a differential eqn from the eqn

Soln  
 a) differential eqn can be defined as a mathematical expression that relates some function with its derivatives. In applications, the functions usually represent physical quantities, the derivatives represent their rates of changes and the equation defines a relationship between the two (eq)

(b)  $y = Ae^{-4x} + Be^{-6x}$  — (\*)

Soln  
 $\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$  — (1)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$  — (2)

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$  (Taking eqn (2))

$\Rightarrow$  multiply through by 6.

$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$

Solve eqn (1) and (2) using elimination method

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$

+  $6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$

$\therefore \frac{d^2y}{dx^2} + \frac{6dy}{dx} = -8Ae^{-4x}$

$\Rightarrow \frac{-1}{8} \left[ \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] = Ae^{-4x}$  (divided through by -8)

divide through by  $e^{-4x}$

$$\frac{-1}{8e^{-4x}} \left[ 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] = A$$

$\Rightarrow$  To get B

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$$

$$4 \frac{dy}{dx} = -16Ae^{-4x} - 24Be^{-6x}$$

Solving eqn (6) and (3) using elimination method

$$4 \frac{dy}{dx} = -16Ae^{-4x} - 24Be^{-6x}$$

$$+ \frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$4 \frac{dy}{dx} + \frac{d^2y}{dx^2} = 12Be^{-6x}$$

$$\frac{1}{12} \left[ 4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] = Be^{-6x}$$

$$\frac{1}{12e^{-6x}} \left[ 4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] = B$$

Putting the value of A & B in eqn (\*)

$$y = \frac{-1}{8e^{-4x}} \left[ \frac{6dy}{dx} + \frac{d^2y}{dx^2} \right] \cdot e^{-4x} + \frac{1}{12e^{-6x}} \left[ 4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] \cdot e^{-6x}$$

$$y = \frac{-1}{8} \left[ \frac{6dy}{dx} + \frac{d^2y}{dx^2} \right] + \frac{1}{12} \left[ 4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right]$$

$$y = \frac{-5}{12} \frac{dy}{dx} - \frac{4}{96} \frac{d^2y}{dx^2}$$

$$y = \frac{-5}{12} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2}$$

• Multiplying through by 24

$$y \times 24 = \frac{-5}{12} \frac{dy}{dx} \cdot \frac{24}{1} - \frac{1}{24} \frac{d^2y}{dx^2} \cdot 24$$

$$24y = -10 \frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$1) \quad y = Ae^{-4x} + Be^{-6x}$$