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C. Chemical Engineering

CH/Engg1/030

Eng 282 Assignment

Assignment 1

If a Define differential equation and give 2 examples.

Differential equation is the relationship between an independent variable x , or dependent variable y and one or more derivative of y with respect to x .

Example: 1) $\frac{dy}{dx} - 3y = x^2$

2) $(x^2 + xy) \frac{dy}{dx} = xy - y^2 \frac{dy}{dx}$

b. i) $y = Ae^{-4x} + Be^{-6x}$

ii) Second order of the differential equation can be formed

iii) This is due to the presence of 2 arbitrary constants A and B.

iv) $y = Ae^{-4x} + Be^{-6x} \quad \text{--- (1)}$

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{--- (2)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (3)}$$

From eq (2) $Ae^{-4x} = y - Be^{-6x} \quad \text{--- (4)}$

Substitute (4) in eq (3)

$$\frac{d^2y}{dx^2} = -4(y - Be^{-6x}) - 6Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4y + 4Be^{-6x} - 6Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4y - 2Be^{-6x} \quad \text{--- (5)}$$

From eq (4) $Be^{-6x} = y - Ae^{-4x} \quad \text{--- (6)}$

Substitute (6) in (5) $\frac{d^2y}{dx^2} = -4Ae^{-4x} - 6(y - Ae^{-4x})$

$$\frac{d^2y}{dx^2} = -4Ae^{-4x} - 6y + 6Ae^{-4x}$$

$$\frac{dy}{dx} = -6y + 2Ae^{-4x} \quad \text{--- (B)}$$

$$\text{from eqn (A): } 2Be^{-6x} = -4y - \frac{dy}{dx} \quad \text{--- (C)}$$

$$\text{from eqn (B): } 2Ae^{-4x} = \frac{dy}{dx} + 6y \quad \text{--- (D)}$$

Substitute (C) and (D) in eqn (3)

$$\frac{d^2y}{dx^2} = 8\left(\frac{dy}{dx} + 6y\right) + 18\left(-4y - \frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = 8\frac{dy}{dx} + 48y - 72y - 18\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -10\frac{dy}{dx} - 24y$$

$$\text{ANS: } \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0.$$