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Mechatronics Engineering
ENG 282

1 a) A differential equation is a relationship between an Independent Variable (x) and dependent Variable (y) and One or more derivative of y with respect to x.

Examples: i) $dy/dx = P + y/x$

ii) $dy/dx = y + y/x$

b) $y = Ae^{-4x} + Be^{-6x}$

c) i) A Second Order differential Equation

ii) A Second Order differential equation Can be formed because it Contains 2 Constants in the degenerate Equation

iii) $y = Ae^{-4x} + Be^{-6x}$

Solution:

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{— (1)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{— (2)}$$

Solving eqn (1) and (2) Simultaneously

Multiply eqn (1) by 6

$$\frac{6dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{— (3)}$$

$$\frac{d^2y}{dx^2} = +16Ae^{-4x} + 36Be^{-6x} \quad \text{— (4)}$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6\,dy/dx + d^2y/dx^2}{-8e^{-4x}} \quad -(5)$$

Substituting eqn (5) into eqn (1)

$$dy/dx = 4\left(\frac{6\,dy/dx + d^2y/dx^2}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\,dy/dx + d^2y/dx^2}{2} - 6Be^{-6x}$$

Multiply through by 2.

$$2\,dy/dx = 6\,dy/dx + d^2y/dx^2 - 12Be^{-6x}$$

$$2\,dy/dx - 6\,dy/dx = d^2y/dx^2 - 12Be^{-6x}$$

$$-4\,dy/dx - d^2y/dx^2 = -12Be^{-6x}$$

$$\frac{-4\,dy/dx - d^2y/dx^2}{-12e^{-6x}} = B \qquad \therefore \frac{4\,dy/dx + d^2y/dx^2}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equation

$$\therefore y = \frac{6\,dy/dx + d^2y/dx^2}{-8e^{-4x}} \times e^{-4x} + \frac{4\,dy/dx + d^2y/dx^2}{+12e^{-6x}} * e^{-6x}$$

$$y = \frac{6\,dy/dx + d^2y/dx^2}{-8} + \frac{4\,dy/dx + d^2y/dx^2}{+12}$$

$$y = \frac{-12\,dy/dx - 12\,d^2y/dx^2 + 32\,dy/dx + 8\,d^2y/dx^2}{96}$$

$$y = \frac{-40\,dy/dx - 4\,d^2y/dx^2}{96}$$

$$96y = -40\,dy/dx - 4\,d^2y/dx^2$$

$$24y = -10\,dy/dx - d^2y/dx^2$$

$$\therefore \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$