

17/EN903/016

## ASSIGNMENT 1

1. Define a differential equation and give two examples

A differential equation is a relationship between an independent variable, a dependent variable,  $Y$ , and one or more derivatives of  $y$  with respect to  $x$

Example: i)  $xy \frac{dy^2}{dx^2} + y \frac{dy}{dx} + e^{2x} = 0$

ii)  $y \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 19x^2 = \sin 4y$

- b. An expression has been obtained for an engineering system to be given in equation (i)

$$Y = Ae^{-4x} + Be^{-6x} \quad \text{--- (i)}$$

- i) What is the order of the differential equation that can be formed from the expression

: second order equation

- ii) Give a reason for your answer in b(i)

It is because ~~the expression contains two terms~~

- iii) Form a differential equation from the expression

$$Y = Ae^{-4x} + Be^{-6x} \quad \text{--- (i)}$$

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{--- (ii)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (iii)}$$

$$\rightarrow 4Ae^{-4x} = -\frac{dy}{dx} - 6Be^{-6x}$$

$$A = \left( -\frac{dy}{dx} - 6Be^{-6x} \right) \cdot \frac{1}{4e^{-4x}} \quad \text{--- (iv)}$$

Substitute (iv) and (iii)

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16 \left( -\frac{dy}{dx} - 6Be^{-6x} \right) \cdot \frac{1}{4e^{-4x}} e^{4x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16 \cdot \left( -\frac{dy}{dx} - \frac{6Be^{-6x}}{4} \right) + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} - 24Be^{-6x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} + 12Be^{-6x}$$

$$B = \left( \frac{d^2y}{dx^2} + 4\frac{dy}{dx} \right) \cdot \frac{1}{12e^{-6x}} \quad \text{--- (v)}$$

substitute eqn (v) and eqn (iv)

$$A = \left( -\frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2} - 6 \left( \frac{d^2y}{dx^2} + \frac{4dy}{dx} \right) \cdot \frac{1}{12e^{-6x}} \right) e^{-6x} \cdot \frac{1}{4e^{-4x}}$$

$$A = \left( -\frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} \right) \cdot \frac{1}{4e^{-4x}}$$

$$A = \left( -\frac{3dy}{dx} - \frac{d^2y}{2dx^2} \right) \cdot \frac{1}{4e^{-4x}} \quad \dots \dots \text{(vi)}$$

Put eqn (vi) and eqn (v) in eqn (v)

$$Y = Ae^{-4x} + Be^{-6x} \quad \dots \dots \text{(i)}$$

$$Y = \left( -\frac{3dy}{dx} - \frac{d^2y}{2dx^2} \right) \cdot \frac{1}{4e^{-4x}} \cdot e^{-4x} + \left( \frac{d^2y}{dx^2} + \frac{4dy}{dx} \right) \cdot \frac{1}{12e^{-6x}} \cdot e^{-6x}$$

$$Y = \frac{1}{4} \left( -\frac{3dy}{dx} - \frac{d^2y}{2dx^2} \right) + \frac{1}{12} \left( \frac{d^2y}{dx^2} + \frac{4dy}{dx} \right)$$

$$Y = -\frac{3dy}{4dx} - \frac{d^2y}{8dx^2} + \frac{d^2y}{12dx^2} + \frac{dy}{3dx}$$

$$Y = -\frac{5dy}{12dx} - \frac{d^2y}{24dx^2}$$

$$Y = \left( \frac{5dy}{dx} - \frac{d^2y}{24dx^2} \right) \cdot \frac{1}{12}$$