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Computer ENG.
17/ENG02/045

A 1a.) A differential equation is a relationship between an independent variable x, a dependent variable y and one ore derivatives of y with respect to x

Example: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} 9y = \sin 2x$

1b.) It is a second order equation

ii.) It is because a function with 2- arbitrary Constants gives a 2nd order equation.

iii.) $y = Ae^{-4x} + Be^{-6x}$ ---------- (I)

$\frac{dy}{dx} = -4Ae^{-4x} + (-6)Be^{-6x}$ ---------- (II)

$\frac{d^2y}{dx} = 16Ae^{-4x} + 36Be^{-6x}$ ---------- (III)

from eqn (I)

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$

$4Ae^{x} = -\frac{dy}{dx} - 6Be^{-6x}$

$A = \left(\frac{-dy}{dx} - 6Be^{-6x}\right) \cdot \frac{1}{4e^{-4x}}$ ---------- (IV)

Substitute eqn (IV) into eqn (III)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$

$$\frac{d^2y}{dx^2} = 16\left(\frac{-dy}{dx} - 6Be^{-6x}\right)e^{-4x}\cdot\frac{1}{16e^{-4x}} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -\frac{4dy}{dx} - 24Be^{-6x} + 36^{-6x}$$

$$\frac{d^2y}{dx^2} = -\frac{4dy}{dx} + 12Be^{-6x}$$

~~$B = \left(\frac{d^2y}{dx^2} + \cdots\right)$~~

$$B = \left(\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right)\cdot\frac{1}{12e^{-6x}} \quad \text{---- (v)}$$

$$A = \left(\frac{dy}{dx} - 6\left(\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right)e^{-6x}\frac{1}{12e^{-6x}}\right)\cdot\frac{1}{4e^{-4x}}$$

$$A = \left(\frac{-dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2} - \frac{2dy}{dx}\right)\cdot\frac{1}{4e^{-4x}}$$

$$A = \left(\frac{-3dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right)\cdot\frac{1}{4e^{-4x}} \quad \text{---- eqn (vi)}$$

Put eqn (v), (vi) into eqn (i)

$$y = Ae^{-4x} + Be^{-6x}$$

$$y = \left(\frac{-3dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right)e^{-4x}\cdot\frac{1}{4e^{-4x}} + \left(\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right)e^{-6x}\cdot\frac{1}{12e^{-6x}}$$

$$y = \frac{3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2} + \frac{1}{12}\frac{d^2y}{dx^2} + \frac{1}{3}\frac{dy}{dx}$$

$$y = \frac{-5}{12}\frac{dy}{dx} - \frac{1}{24}\frac{d^2y}{dx^2}$$

$$y = \left(\frac{-5}{m} \frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2} \right) \cdot \frac{1}{2}$$