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Mechatronics Engineering
ENG 282

1a. A differential equation is a relationship between an independent variable (x) and dependent variable(y) and one or more derivatives of y with respect to x.

Examples: (i) $dy/dx = 4 + y/x$

(ii) $dy/dx = y + y/x$

b. $y = Ae^{-4x} + Be^{-6x}$

i. A Second order differential equation

ii. A Second order differential equation can be formed because it contains 2 constants in the general equation

iii $y = Ae^{-4x} + Be^{-6x}$

Solution

$dy/dx = -4Ae^{-4x} - 6Be^{-6x}$ .... (1)

$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x}$ .... (2)

Solving eqn(i) and (ii) simultaneously

multiply eqn (i) by 6

$$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \ldots \text{(iii)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \ldots \text{(iv)}$$

eqn (1) + (2)

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6\,{dy}/{\partial x} + {\partial^2 y}/{\partial x^2}}{-8e^{-4x}} \quad \ldots \text{(v)}$$

Subst eqn (v) into eqn(1)

$$\frac{\partial y}{\partial x} = 4\left(\frac{6\,{dy}/{dx} + {d^2y}/{dx^2}}{-8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$= \frac{6\,{dy}/{\partial x} + {\partial^2 y}/{\partial x^2}}{2} - 6Be^{-6x}$$

$$2\frac{\partial y}{\partial x} = \frac{6\,{\partial y}/{\partial x} + {\partial^2 y}/{\partial x^2}}{\cancel{2}} - 12Be^{-6x}$$

$$2\frac{\partial y}{\partial x} - 6\frac{\partial y}{\partial x} = \frac{\partial^2 y}{\partial x^2} - 12Be^{-6x}$$

$$-4\frac{dy}{\partial x} - \frac{d^2y}{\partial x^2} = -12 \cdot Be^{-6x}$$

$$\frac{-4\,{dy}/{\partial x} - {\partial^2 y}/{\partial x^2}}{-12e^{-6x}} = B \quad \therefore \frac{4\,{dy}/{\partial x} + {\partial^2 y}/{\partial x^2}}{12e^{-6x}} = B$$

Subst. A and B into the degenerate equation

$$\therefore y = \frac{6\,\partial y/\partial x + d^2y/\partial x^2 \times \cancel{e^{-4x}}}{-8\cancel{e^{-4x}}} + \frac{4\,\partial y/\partial x + \partial^2 y/\partial x^2 \times \cancel{e^{-6x}}}{12\cancel{e^{-6x}}}$$

$$y = \frac{6\,dy/\partial x + \partial^2 y/\partial x^2}{-8} + \frac{4\,\partial y/\partial x + \partial^2 y/\partial x^2}{12}$$

$$y = \frac{-72\,dy/\partial x - 12\,d^2y/\partial x^2 + 32\,dy/\partial x + 8\,\partial^2 y/\partial x^2}{96}$$

$$y = \frac{-40\,dy/\partial x - 4\,\partial^2 y/\partial x^2}{96}$$

$$96y = -40\frac{dy}{dx} - 4\frac{d^2y}{\partial x^2}$$

$$24y = -10\frac{\partial y}{\partial x} - \frac{d^2y}{\partial x^2}$$

$$\therefore \frac{\partial^2 y}{\partial x^2} + 10\frac{\partial y}{\partial x} + 24y = 0$$

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