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1a A differential equation is a relationship between an independent variable 'x' and dependent variable 'y' and one or more derivative of y with respect to x.
Eg; (i) $\frac{dy}{dx} = 2 + y/x$
(ii) $\frac{dy}{dx} = y + y/x$

b $y = Ae^{-4x} + Be^{-6x}$

(i) A second order differential equation.

(ii) This is because it contains two variables

(iii) $y = Ae^{-4x} + Be^{-6x}$

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{— i}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{— ii}$$

Solving equation (i) and (ii) simultaneously
Multiply eqn (i) by 6

$$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{— iii}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{— iv}$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = 6\,dy/dx + \frac{d^2y}{dx^2} \quad \text{— v}$$

Substituting eqn (v) into eqn (i)

$$\frac{dy}{dx} = 4\left(\frac{6\,dy/dx + d^2y/dx^2}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\,dy/dx + d^2y/dx^2}{2} - 6Be^{-6x}$$

Multiply through by 2

$$\frac{2dy}{dx} = \frac{6dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$\frac{2dy}{dx} - \frac{6dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-\frac{4dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\,{}^{dy}/_{dx} - {}^{d^2y}/_{dx^2}}{-12e^{-6x}} = B \qquad \therefore B = \frac{4\,{}^{dy}/_{dx} + {}^{d^2y}/_{dx^2}}{12e^{-6x}}$$

Substitute A and B into the degenerate equation.

$$\therefore y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \times e^{-4x} + \frac{4\,{}^{dy}/_{dx} + {}^{d^2y}/_{dx^2}}{+12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6\,{}^{dy}/_{dx} + {}^{d^2y}/_{dx^2}}{-8} + \frac{4\,{}^{dy}/_{dx} + {}^{d^2y}/_{dx^2}}{+12}$$

$$y = \frac{+18\frac{dy}{dx} + 3\frac{d^2y}{dx^2} - 8\,{}^{dy}/_{dx} - 2\,{}^{d^2y}/_{dx^2}}{-24}$$

$$y = \frac{10\,{}^{dy}/_{dx} + \frac{d^2y}{dx^2}}{-24}$$

$$-24y = 10\,{}^{dy}/_{dx} + \frac{d^2y}{dx^2}$$

$$10\frac{dy}{dx} + \frac{d^2y}{dx^2} + 24y = 0$$

$$\therefore \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$

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