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1a A differential equation is a relationship between an independent variable x and dependent variable "y" and one or more derivative of y with respect to x

E.g; (i) $\frac{dy}{dx} = 2 + \frac{y}{x}$

(ii) $\frac{dy}{dx} = y + \frac{y}{x}$

b. $y = Ae^{-4x} + Be^{-6x}$

(i) A second order differential equation

ii This is because it contains two variables

iii $y = Ae^{-4x} + Be^{-6x}$

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{---(i)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{---(ii)}$$

(Solving equation (i) and (ii) simultaneously)

Multiply eqn(i) by 6.

$$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{---(iii)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{---(iv)}$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} \quad \text{---(v)}$$

Substituting eqn(v) into eqn(i)

$$\frac{dy}{dx} = 4\left(\frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

Multiply through by 2.

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B$$

$$\therefore B = \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}}$$

Substitute A and B into the degenerate equation.

$$\therefore y = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \times e^{-4x} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12}$$

$$y = \frac{+18 \frac{dy}{dx} + 3 \frac{d^2y}{dx^2} - 8 \frac{dy}{dx} - 2 \frac{d^2y}{dx^2}}{-24}$$

$$y = \frac{10 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-24}$$

Cross Multiply

$$-24y = 10 \frac{dy}{dx} + \frac{d^2y}{dx^2}$$

$$10 \frac{dy}{dx} + \frac{d^2y}{dx^2} + 24y = 0$$

$$\therefore \frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0$$