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1a) A differential equation is a relationship btw an independent variable 'x' and dependent variable 'y' and one or more derivate of y with respect to x

Eg: (i) $dy/dx = 2 + y/x$

(ii) $dy/dx = y + y/x$

b) $y = Ae^{-4x} + Be^{-6x}$

i) A second order differential equation.

ii) This is because it contains two variables

iii) $y = Ae^{-4x} + Be^{-6x}$

solution

$$dy/dx = -4Ae^{-4x} - 6Be^{-6x} \quad \text{---------- (i)}$$

$$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x} \quad \text{---------- (ii)}$$

Solving equation (i) & (ii) simultaneously

Multiply equ (i) by 6

$$6dy/dx = -24Ae^{-4x} - 36Be^{-6x} \quad \text{---------- (iii)}$$

$$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x} \quad \text{---------- (iv)}$$

$$6dy/dx + d^2y/dx^2 = -8Ae^{-4x}$$

$$\therefore A = 6dy/dx + d^2y/dx^2 \quad \text{---------- (v)}$$

Substituting equ (v) into equ (ii)

$$dy/dx = A\left(\frac{6dy/dx + d^2y/dx^2}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$dy/dx = \frac{6dy/dx + d^2y/dx^2}{2} - 6Be^{-6x}$$

Multiply through by 2.

$$2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2\frac{dy}{dx} - 6\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B \qquad \therefore B = \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}}$$

Substitute A & B into the ~~exp~~ degenerate equation

$$\therefore y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \times e^{-4x} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12}$$

$$y = \frac{+18\frac{dy}{dx} + 3\frac{d^2y}{dx^2} - 8\frac{dy}{dx} - 2\frac{d^2y}{dx^2}}{-24}$$

$$y = \frac{10\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-24}$$

$$-24y = 10\frac{dy}{dx} + \frac{d^2y}{dx^2}$$

$$10\frac{dy}{dx} + \frac{d^2y}{dx^2} + 24y = 0$$

$$\therefore \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$