Given that: $F = x^2 i + (3x+2) j + \sin x k$

Find (a) $\frac{dF}{dx}$ (b) $\frac{d^2F}{dx^2}$ (c) $\left|\frac{dF}{dx}\right|$ (d) $\frac{d}{dx}(F.F)$ at $x=1$

If $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t} k$ determine.

(a) $\frac{dr}{dt}$ (b) $\frac{d^2r}{dt^2}$ (c) the value of $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

Solution.

If $x^2 i + (3x+2) j + \sin x k$.

(a) $\frac{dF}{dx} = 2x i + 3j + \cos x k$ at $x=1$, $\frac{dF}{dx} = 2i + 3j + k$

(b) $\frac{d^2F}{dx^2} = 2i + 0j - \sin x k$ at $x=1$; $\frac{d^2F}{dx^2} = 2i + 0j + 0.0175k$

(c) $\left|\frac{dF}{dx}\right| = \sqrt{2^2+3^2+1^2} = \sqrt{4+9+1} = \sqrt{14}$

(d) $\frac{d}{dx}(F.F) = (F.F) = x^2 + 9x^2 + 12x + 4 + \sin^2 x$

$\frac{d}{dx}(F.F) = 4x^3 + 18x + 12 + 2\sin x \cos x$ at $x=1$

$\frac{d}{dx}(F.F) = 4 + 18 + 12 + 0.035 = 34.035$

$r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t} k$

(a) $\frac{dr}{dF} = (2t+3)i - 6(\cos 3t) j + 6e^{2t} k$

$\frac{d^2r}{dt^2} = 2i + (18\sin 3t) j + 12e^{2t} k$

$\left|\frac{d^2r}{dt^2}\right| = 2i + 0j + 12k$

$= \sqrt{2^2+12} = \sqrt{148} = 2\sqrt{37}$.

at $t=0$