

QUESTION 1 [20 MARKS]

(a) Using an initial guess vector of $m_0 = [0;0;0]$, determine the values of the variables in Equation (1) using Jacobi iterative method manually showing only three iterations.

(b) Write a MATLAB mfile program to solve the problem in 1(a) and tabulate the results showing the number of iterations, the corresponding values of the variables and the error, which is calculated from the norms. Take the tolerance of the error to be $1E-15$.

SOLUTION A. MANUAL SOLUTION

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A. Using an initial guess vector of $m_0 = [0;0;0]$, determine the values of the variables in Equation 1. Using Jacobi iterative method manually showing only three iterations.

Soln.

$$\begin{cases} 10m_1 - 2m_2 + m_3 = 9 & \text{--- (i)} \\ +2m_1 + 10m_2 - 2m_3 = 12 & \text{--- (ii)} \\ -2m_1 - 5m_2 + 10m_3 = 18 & \text{--- (iii)} \end{cases}$$

For equation (i),

$$10m_1 - 2m_2 + m_3 = 9$$

$$10m_1 = 2m_2 - m_3 + 9$$

$$m_1 = \frac{2m_2}{10} - \frac{m_3}{10} + \frac{9}{10}$$

$$m_1 = 0.2m_2 - 0.1m_3 + 0.9$$

For equation (ii),

$$-2m_1 + 10m_2 - 2m_3 = 12$$

$$10m_2 = 2m_1 + 2m_3 + 12$$

$$m_2 = \frac{2}{10}m_1 + \frac{2}{10}m_3 + \frac{12}{10}$$

$$\therefore m_2 = 0.2m_1 + 0.2m_3 + 1.2$$

For equation (iii),

$$-2m_1 - 5m_2 + 10m_3 = 18$$

$$10m_3 = 2m_1 + 5m_2 + 18$$

$$m_3 = \frac{2}{10}m_1 + \frac{5}{10}m_2 + \frac{18}{10}$$

$$m_3 = 0.2m_1 + 0.5m_2 + 1.8$$

Therefore

$$\begin{aligned} m_1 &= 0m_1 + 0.2m_2 - 0.1m_3 + 0.9 \\ m_2 &= 0.2m_1 + 0m_2 + 0.2m_3 + 1.2 \\ m_3 &= 0.2m_1 + 0.5m_2 + 0m_3 + 1.8 \end{aligned}$$

In matrix form;

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

For iteration $i=0$, using $m_0 = [0; 0; 0]$

$$\begin{bmatrix} m_1(i+1) \\ m_2(i+1) \\ m_3(i+1) \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} m_1(i) \\ m_2(i) \\ m_3(i) \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

Hence

$$\begin{bmatrix} m_1(i+1) \\ m_2(i+1) \\ m_3(i+1) \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

$$\text{Norm}_b = \sqrt{0^2 + 0^2 + 0^2} = 0$$

$$\text{Norm}_a = \sqrt{0.9^2 + 1.2^2 + 1.8^2} = 2.343075$$

$$\text{Error} = \text{Norm}_a - \text{Norm}_b = 2.343075 - 0 = 2.343075$$

For $i=1$

$$\begin{bmatrix} m_1(i+1) \\ m_2(i+1) \\ m_3(i+1) \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.96 \\ 1.74 \\ 2.58 \end{bmatrix}$$

$$m_1(i+1) = 0 \times 0.9 + 0.2 \times 1.2 + (-0.1) \times 1.8 + 0.9 = 0.96$$

$$m_2(i+1) = 0.2 \times 0.9 + 0 \times 1.2 + 0.2 \times 1.8 + 1.2 = 1.74$$

$$m_3(i+1) = 0.2 \times 0.9 + 0.5 \times 1.2 + 0 \times 1.8 + 1.8 = 2.58$$

$$\text{Norm}_b = \sqrt{0.9^2 + 1.2^2 + 1.8^2} = 2.343075$$

$$\text{Norm}_a = \sqrt{0.96^2 + 1.74^2 + 2.58^2} = 3.256624$$

$$\text{Error} = \text{Norm}_a - \text{Norm}_b = 3.256624 - 2.343075 = 0.913549$$

$$\text{For } i = 2$$

$$\begin{bmatrix} M_1(i+1) \\ M_2(i+1) \\ M_3(i+1) \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.96 \\ 1.74 \\ 2.58 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.99 \\ 1.908 \\ 2.862 \end{bmatrix}$$

$$M_1(i+1) = 0 \times 0.96 + 0.2 \times 1.74 - 0.1 \times 2.58 + 0.9 = 0.99$$

$$M_2(i+1) = 0.2 \times 0.96 + 0 \times 1.74 + 0.2 \times 2.58 + 1.2 = 1.908$$

$$M_3(i+1) = 0.2 \times 0.96 + 0.5 \times 1.74 + 0 \times 2.58 + 1.8 = 2.862$$

$$\text{Norm } b = \sqrt{0.96^2 + 1.74^2 + 2.58^2} = 3.256624$$

$$\text{Norm } a = \sqrt{0.99^2 + 1.908^2 + 2.862^2} = 3.579331$$

$$\text{Error} = \text{Norm } a - \text{Norm } b = 3.579331 - 3.256624 = \underline{\underline{0.322707}}$$

For $i = 3$

$$\begin{bmatrix} M_1(i+1) \\ M_2(i+1) \\ M_3(i+1) \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.99 \\ 1.908 \\ 2.862 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.9954 \\ 1.9704 \\ 2.952 \end{bmatrix}$$

$$M_1(i+1) = 0 \times 0.99 + 0.2 \times 1.908 - 0.1 \times 2.862 + 0.9 = 0.9954$$

$$M_2(i+1) = 0.2 \times 0.99 + 0 \times 1.908 + 0.2 \times 2.862 + 1.2 = 1.9704$$

$$M_3(i+1) = 0.2 \times 0.99 + 0.5 \times 1.908 + 0 \times 2.862 + 1.8 = 2.952$$

$$\text{Norm } b = \sqrt{0.99^2 + 1.908^2 + 2.862^2} = 3.579331$$

$$\text{Norm } a = \sqrt{0.9954^2 + 1.9704^2 + 2.952^2} = 3.686136$$

$$\text{Error} = \text{Norm } a - \text{Norm } b = 3.686136 - 3.579331 = \underline{\underline{0.106806}}$$

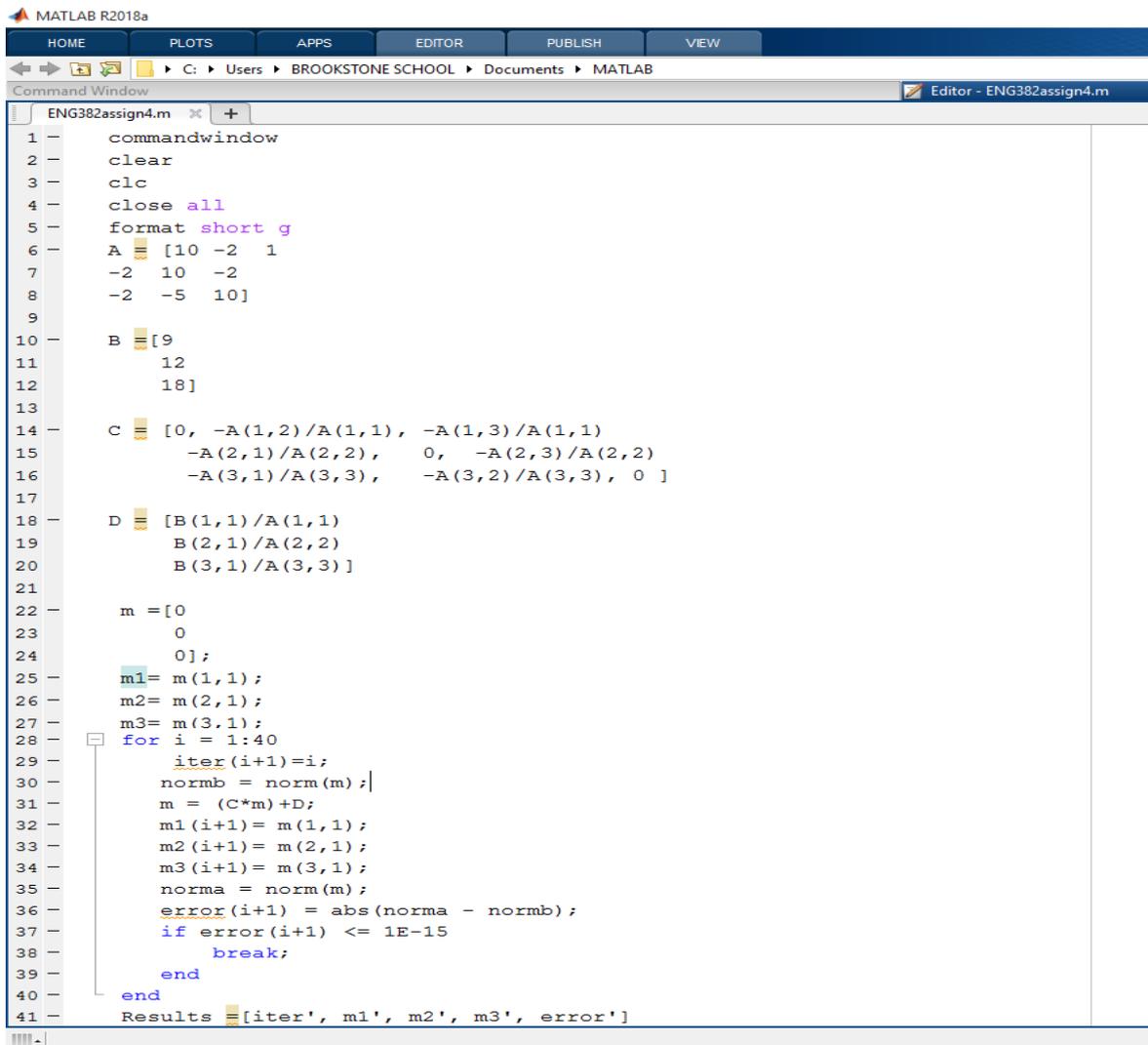
Hence, The approximate results of the iteration using Jacobi iterative method is:

$$M_1 \approx 1, M_2 \approx 2 \text{ and } M_3 \approx 3$$

$$M_{i+1} = [1; 2; 3]$$

FIGURE A. MANUAL SOLUTION

SOLUTION B. MATLAB SOLUTION

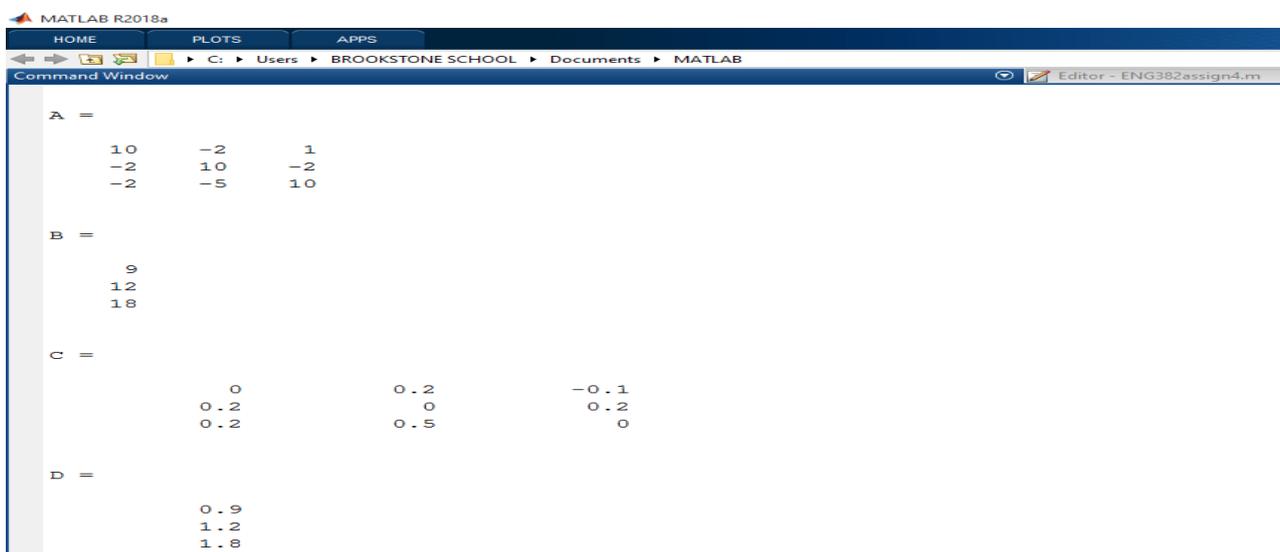


```

1 clearwindow
2 clear
3 clc
4 close all
5 format short g
6 A = [10 -2 1
7      -2 10 -2
8      -2 -5 10]
9
10 B = [9
11      12
12      18]
13
14 C = [0, -A(1,2)/A(1,1), -A(1,3)/A(1,1)
15      -A(2,1)/A(2,2), 0, -A(2,3)/A(2,2)
16      -A(3,1)/A(3,3), -A(3,2)/A(3,3), 0 ]
17
18 D = [B(1,1)/A(1,1)
19      B(2,1)/A(2,2)
20      B(3,1)/A(3,3)]
21
22 m = [0
23      0
24      0];
25 m1 = m(1,1);
26 m2 = m(2,1);
27 m3 = m(3,1);
28 for i = 1:40
29     iter(i+1) = i;
30     normb = norm(m);
31     m = (C*m) + D;
32     m1(i+1) = m(1,1);
33     m2(i+1) = m(2,1);
34     m3(i+1) = m(3,1);
35     norma = norm(m);
36     error(i+1) = abs(norma - normb);
37     if error(i+1) <= 1E-15
38         break;
39     end
40 end
41 Results = [iter', m1', m2', m3', error']

```

FIGURE B1: CODE FOR THE SOLUTION



```

A =
    10    -2     1
    -2    10    -2
    -2    -5    10

B =
     9
    12
    18

C =
     0     0.2   -0.1
     0.2     0     0.2
     0.2     0.5     0

D =
     0.9
     1.2
     1.8

iter =
    40

m1 =
     0.9

m2 =
     1.2

m3 =
     1.8

error =
     0

```

```
Results =
```

0	0	0	0	0
1	0.9	1.2	1.8	2.3431
2	0.96	1.74	2.58	0.91355
3	0.99	1.908	2.862	0.32271
4	0.9954	1.9704	2.952	0.10681
5	0.99888	1.9895	2.9843	0.036997
6	0.99947	1.9966	2.9945	0.012185
7	0.99987	1.9988	2.9982	0.0042271
8	0.99994	1.9996	2.9994	0.0013884
9	0.99999	1.9999	2.9998	0.0004829
10	0.99999	2	2.9999	0.00015816
11	1	2	3	5.5172e-05
12	1	2	3	1.8013e-05
13	1	2	3	6.3043e-06
14	1	2	3	2.0512e-06
15	1	2	3	7.2049e-07
16	1	2	3	2.3354e-07
17	1	2	3	8.2356e-08
18	1	2	3	2.6584e-08
19	1	2	3	9.4157e-09
20	1	2	3	3.0253e-09
21	1	2	3	1.0767e-09
22	1	2	3	3.4421e-10
23	1	2	3	1.2315e-10
24	1	2	3	3.9152e-11
25	1	2	3	1.409e-11
26	1	2	3	4.4516e-12
27	1	2	3	1.6125e-12
28	1	2	3	5.0626e-13
29	1	2	3	1.843e-13
30	1	2	3	5.7732e-14
31	1	2	3	2.1316e-14
32	1	2	3	6.2172e-15
33	1	2	3	2.6645e-15
34	1	2	3	8.8818e-16

fx >>

FIGURE B2: RESULTS OF THE CODE