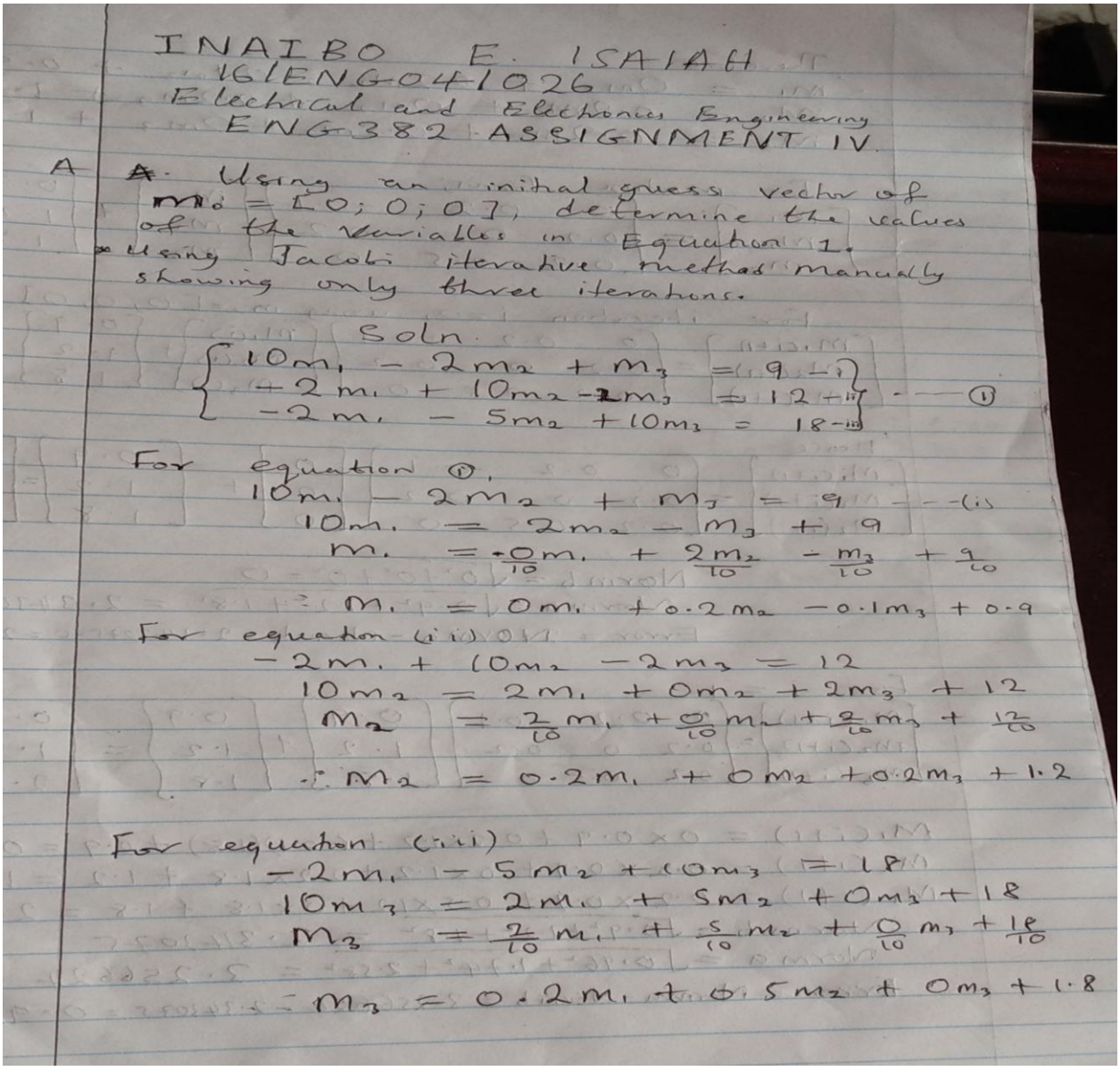
QUESTION 1 [20 MARKS]

(a) Using an initial guess vector of $m_0 = [0;0;0]$, determine the values of the variables in Equation (1) using Jacobi iterative method manually showing only three iterations.

(b) Write a MATLAB mfile program to solve the problem in 1(a) and tabulate the results showing the number of iterations, the corresponding values of the variables and the error, which is calculated from the norms. Take the tolerance of the error to be 1E-15.

SOLUTION A. MANUAL SOLUTION



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ENG382 ASSIGNMENT IV.

A. Using an initial guess vector of $m_0 = [0;0;0]$, determine the values of the variables in Equation 1.
Using Jacobi iterative method manually showing only three iterations.

Soln.

$$\begin{cases} 10m_1 - 2m_2 + m_3 = 9 \text{ --- (i)} \\ +2m_1 + 10m_2 - 2m_3 = 12 \text{ --- (ii)} \\ -2m_1 - 5m_2 + 10m_3 = 18 \text{ --- (iii)} \end{cases} \text{ --- (1)}$$

For equation (1),

$10m_1 - 2m_2 + m_3 = 9$ --- (i)

$10m_1 = 2m_2 - m_3 + 9$

$m_1 = \frac{0}{10}m_1 + \frac{2}{10}m_2 - \frac{1}{10}m_3 + \frac{9}{10}$

$\therefore m_1 = 0m_1 + 0.2m_2 - 0.1m_3 + 0.9$

For equation (ii)

$-2m_1 + 10m_2 - 2m_3 = 12$

$10m_2 = 2m_1 + 0m_2 + 2m_3 + 12$

$m_2 = \frac{2}{10}m_1 + \frac{0}{10}m_2 + \frac{2}{10}m_3 + \frac{12}{10}$

$\therefore m_2 = 0.2m_1 + 0m_2 + 0.2m_3 + 1.2$

For equation (iii)

$-2m_1 - 5m_2 + 10m_3 = 18$

$10m_3 = 2m_1 + 5m_2 + 0m_3 + 18$

$m_3 = \frac{2}{10}m_1 + \frac{5}{10}m_2 + \frac{0}{10}m_3 + \frac{18}{10}$

$\therefore m_3 = 0.2m_1 + 0.5m_2 + 0m_3 + 1.8$

Therefore

$$m_1 = 0m_1 + 0.2m_2 - 0.1m_3 + 0.9$$
$$m_2 = 0.2m_1 + 0m_2 + 0.2m_3 + 1.2$$
$$m_3 = 0.2m_1 + 0.5m_2 + 0m_3 + 1.8$$

In matrix form;

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

For iteration $i=0$, using $m_0 = [0; 0; 0]$

$$\begin{bmatrix} m_{1(i+1)} \\ m_{2(i+1)} \\ m_{3(i+1)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} m_{1(i)} \\ m_{2(i)} \\ m_{3(i)} \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

Hence

$$\begin{bmatrix} m_{1(i+1)} \\ m_{2(i+1)} \\ m_{3(i+1)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$$

$$\text{Norm } b = \sqrt{0^2+0^2+0^2} = 0$$
$$\text{Norm } a = \sqrt{0.9^2+1.2^2+1.8^2} = 2.343075$$
$$\text{Error} = \text{Norm } a - \text{Norm } b = 2.343075 - 0 = 2.343075$$

For $i = 1$

$$\begin{bmatrix} m_{1(i+1)} \\ m_{2(i+1)} \\ m_{3(i+1)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} = \begin{bmatrix} 0.96 \\ 1.74 \\ 2.58 \end{bmatrix}$$

$$M_{1(i+1)} = 0 \times 0.9 + 0.2 \times 1.2 + (-0.1)(1.8) + 0.9 = 0.96$$
$$M_{2(i+1)} = 0.2 \times 0.9 + 0 \times 1.2 + 0.2 \times 1.8 + 1.2 = 1.74$$
$$M_{3(i+1)} = 0.2 \times 0.9 + 0.5 \times 1.2 + 0 \times 1.8 + 1.8 = 2.58$$
$$\therefore \text{Norm } b = \sqrt{0.9^2+1.2^2+1.8^2} = 2.343075$$
$$\text{Norm } a = \sqrt{0.96^2+1.74^2+2.58^2} = 3.256624$$
$$\text{Error} = \text{Norm } a - \text{Norm } b = 3.256624 - 2.343075 = 0.913549$$

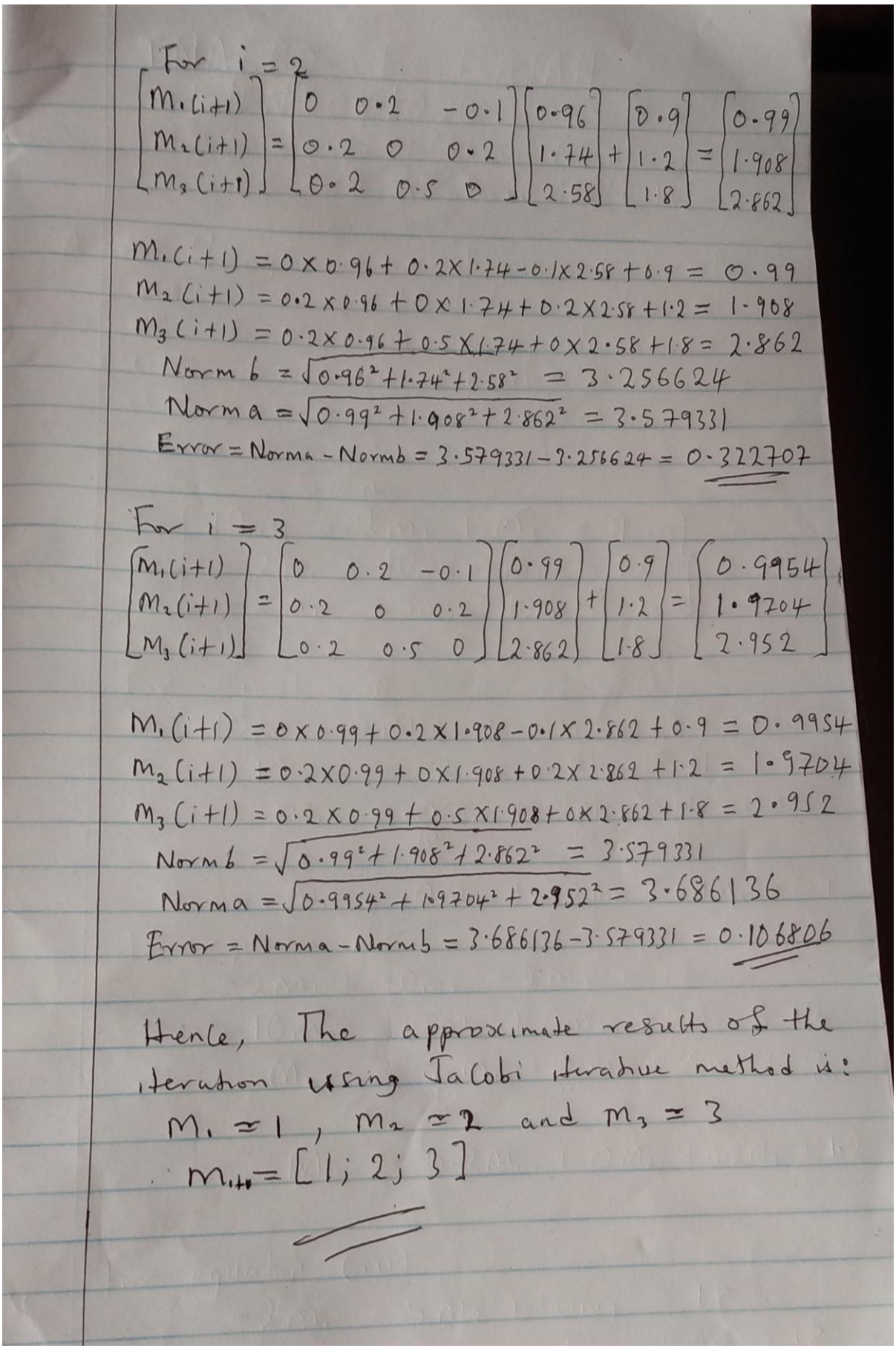


FIGURE A. MANUAL SOLUTION

SOLUTION B. MATLAB SOLUTION

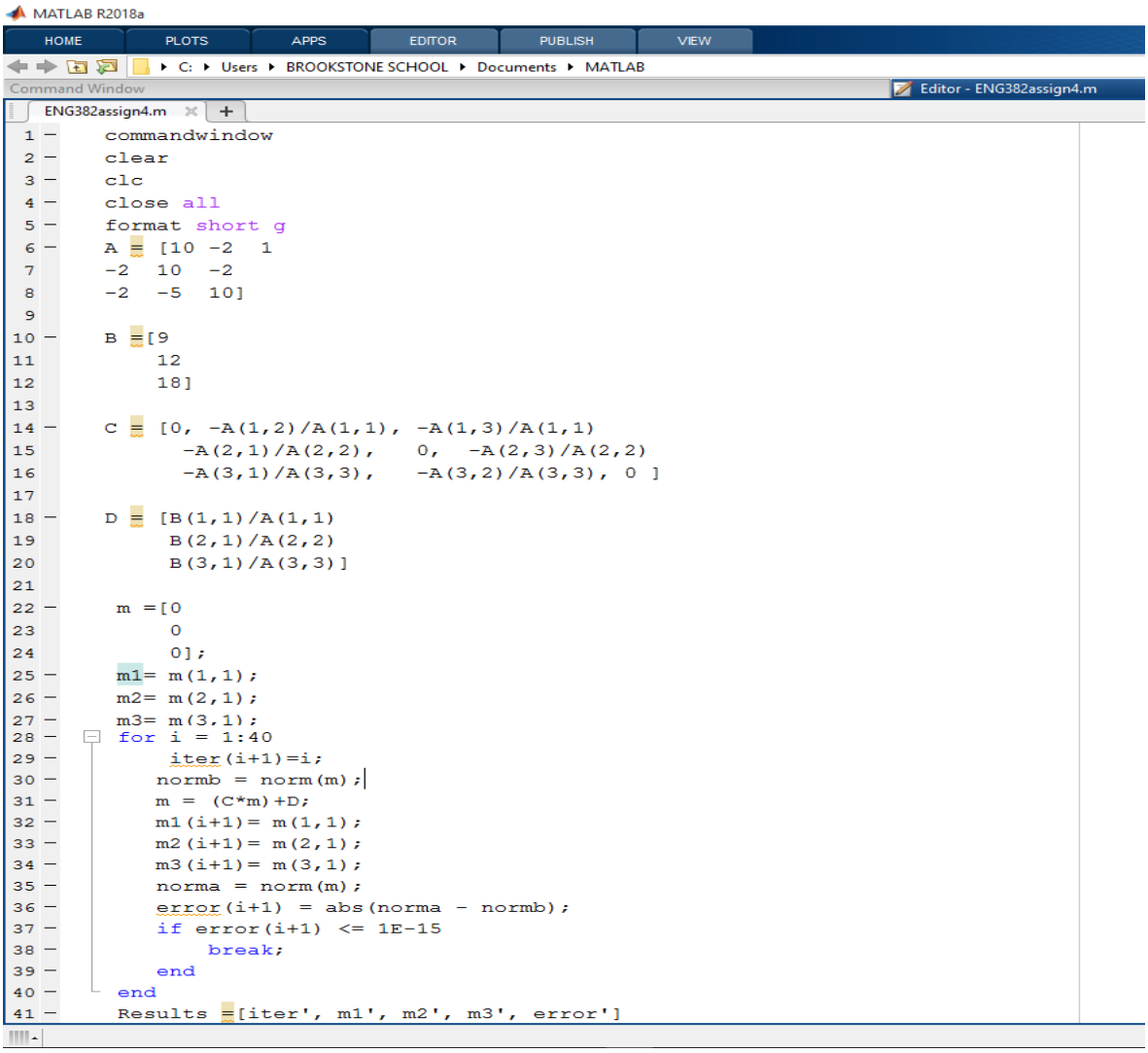
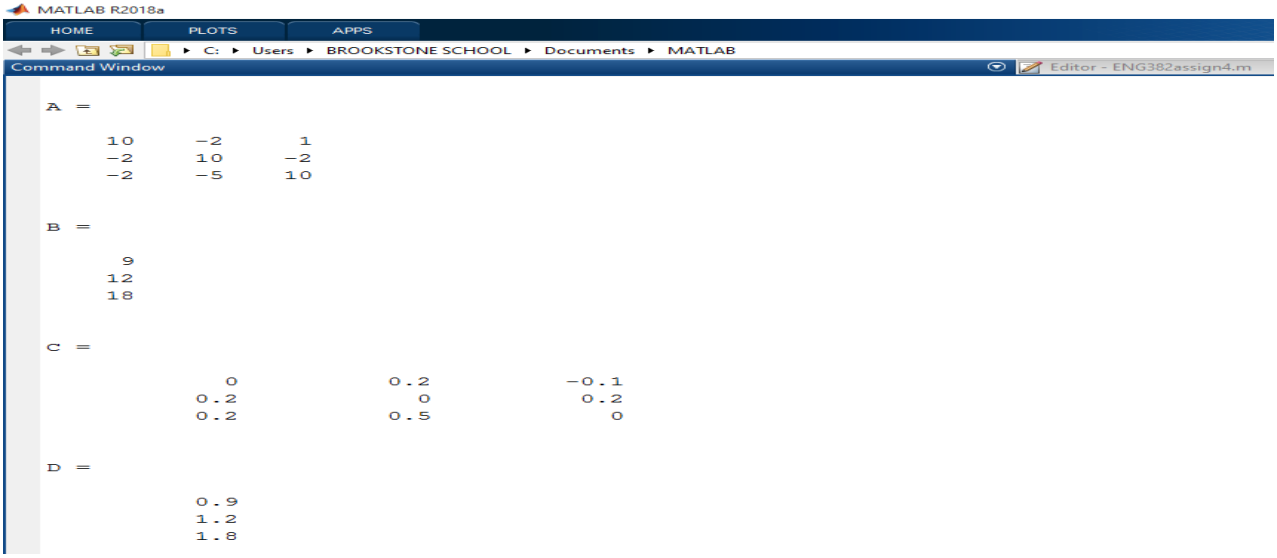


FIGURE B1: CODE FOR THE SOLUTION



```
Results =

            0            0            0            0            0
            1          0.9          1.2          1.8       2.3431
            2         0.96         1.74         2.58      0.91355
            3         0.99        1.908        2.862      0.32271
            4       0.9954       1.9704        2.952      0.10681
            5      0.99888       1.9895       2.9843     0.036997
            6      0.99947       1.9966       2.9945     0.012185
            7      0.99987       1.9988       2.9982    0.0042271
            8      0.99994       1.9996       2.9994    0.0013884
            9      0.99999       1.9999       2.9998    0.0004829
           10      0.99999            2       2.9999   0.00015816
           11            1            2            3   5.5172e-05
           12            1            2            3   1.8013e-05
           13            1            2            3   6.3043e-06
           14            1            2            3   2.0512e-06
           15            1            2            3   7.2049e-07
           16            1            2            3   2.3354e-07
           17            1            2            3   8.2356e-08
           18            1            2            3   2.6584e-08
           19            1            2            3   9.4157e-09
           20            1            2            3   3.0253e-09

           21            1            2            3   1.0767e-09
           22            1            2            3   3.4421e-10
           23            1            2            3   1.2315e-10
           24            1            2            3   3.9152e-11
           25            1            2            3    1.409e-11
           26            1            2            3   4.4516e-12
           27            1            2            3   1.6125e-12
           28            1            2            3   5.0626e-13
           29            1            2            3    1.843e-13
           30            1            2            3   5.7732e-14
           31            1            2            3   2.1316e-14
           32            1            2            3   6.2172e-15
           33            1            2            3   2.6645e-15
           34            1            2            3   8.8818e-16

fx >>
```

FIGURE B2: RESULTS OF THE CODE