OLADIPO TOMI ISAAC
17/ENG05/030
MECHATRONICS ENGINEERING

1a) A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivative of y with respect to x.

E.g (i) $\frac{dy}{dx} = y + \frac{y}{x}$

b) $y = Ae^{-4x} + Be^{-6x}$

i) A Second order differential equation.

ii) This is because it contains two variables.

iii) $y = Ae^{-4x} + Be^{-6x}$

Solution:

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \ldots (1)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \ldots (2)$$

Solve eqn (1) and (2) simultaneously

Multiply eqn (1) by 6

$$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \ldots (3)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \ldots (4)$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} \quad \ldots (5)$$

Subs. eqn (5) into eqn (1)

$$\frac{dy}{dx} = 4\left(\frac{6\frac{dy}{dx} + d^2y/dx^2}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\frac{dy}{dx} + d^2y/dx^2}{2} - 6Be^{-6x}$$

Multiply through by 2

$$2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2\frac{dy}{dx} - 6\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B$$

$$\therefore B = \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}}$$

Sub. A and B into the degenerate equation.

$$\therefore y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \times e^{-4x} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6\frac{dy}{dx} + d^2y/dx^2}{-8} + \frac{4\,dy/dx + d^2y/dx^2}{12}$$

$$y = \frac{18\frac{dy}{dx} + 3\frac{d^2y}{dx^2} - 8\,dy/dx - 2\,d^2y/dx^2}{-24}$$

$$y = \frac{10\,dy/dx + \frac{d^2y}{dx^2}}{-24}$$

$$-24y = 10\frac{dy}{dx} + d^2y/dx^2$$

$$\therefore \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0.$$